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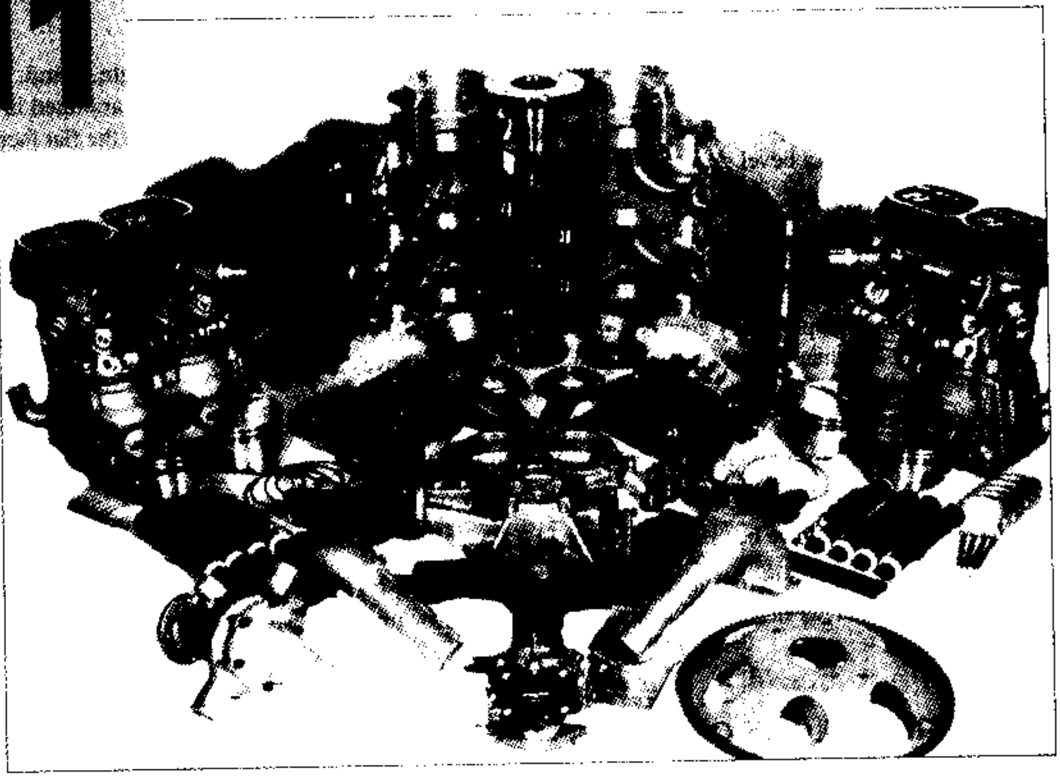


GEARS

11.1 INTRODUCTION

A gear may be defined as any toothed member designed to transmit or receive motion from another member by successively engaging tooth. The gears give a positive drive and provide many advantages over friction drives like belts, ropes and friction drums. They are used in metal cutting machine tools, automobiles, tractors, hoisting and transporting machinery, rolling mills, etc. However, they require special equipment for manufacture. The error in tooth meshing may cause undesirable vibration and noise during operation. The smaller gear is called the *pinion* and the bigger one the *gear wheel*.

11.2 TYPES OF GEARS

The gears may be classified as follows:

Spur gears A spur gear is a cylindrical gear whose tooth traces are straight line generators of the reference cylinder.

Helical gears The helical gear is similar to the spur gear in which the tooth traces are helices.

Double helical (or herringbone) gears A double helical gear is a cylindrical gear in which a part of the face width is right hand and the other left hand, with or without a gap between them.

Spiral gears In spiral gears, the tooth traces are curved lines other than helices.

Bevel gears In bevel gears, the reference surface is a cone. Bevel gears may be straight, spiral, zerol or face gears. In zerol bevel gears, the teeth are curved in the lengthwise direction and are arranged in such a manner that the effective spiral angle is zero. In face gears, the bevel gear teeth are cut on the flat face of the blank. A crown gear is a bevel gear with a reference cone angle of 90°.

Hypoid gears Hypoid gears are similar to the spiral bevel gears with the difference that the axes of the shafts do not intersect.

Worm gears In worm gears, the worm has screw threads and worm wheel teeth.

Planetary gears The planetary gear is a gear pair or a gear train one of whose axes, instead of being fixed in position in the mechanism of which the gear pair is a part, moves around it.

Gears may also be classified based on the orientation of the shafts as:

1. Parallel shafts-spur, helical and double helical gears
2. Intersecting shafts-straight bevel, spiral bevel, zerol bevel and face gears, and
3. Non-parallel and non-intersecting-spiral, hypoid and worm gears.

Some of these gears have been shown in Fig.11.1.

Gears may be of the external, internal and rack and pinion type. In external gears, the teeth of gears mesh externally, whereas in internal gears the teeth of the two gears mesh internally. A rack is a gear of infinite radius.

11.3 TERMINOLOGY OF GEARS

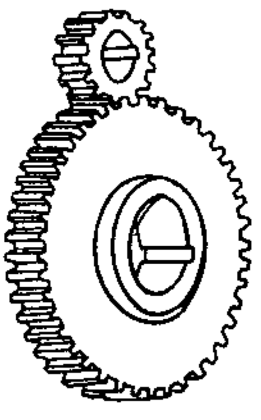
A gear pair in mesh is shown in Fig.11.2. The various terms have been indicated in this figure.

Pitch circle The pitch circle is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

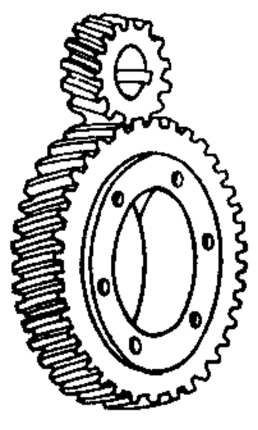
Pitch circle diameter (d) The pitch circle diameter is the diameter of a circle which by pure rolling action would produce the same motion as the toothed gear wheel.

Base circle The base circle is the circle from which involute form is generated.

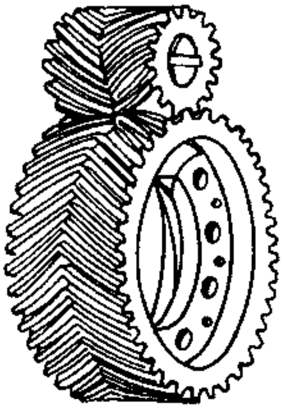
Pitch surface The pitch surface is the surface of the disc which the toothed gear has replaced at the pitch circle.



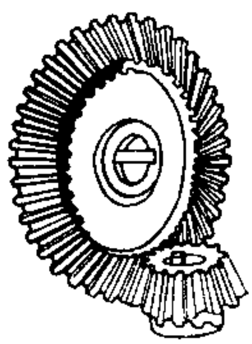
(a) Spur gears



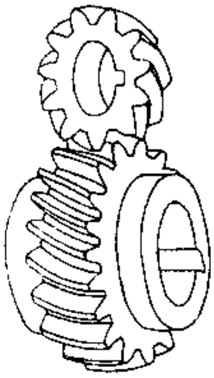
(b) Helical gears



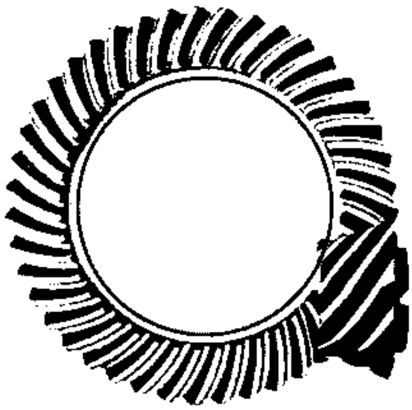
(c) Herringbone gears



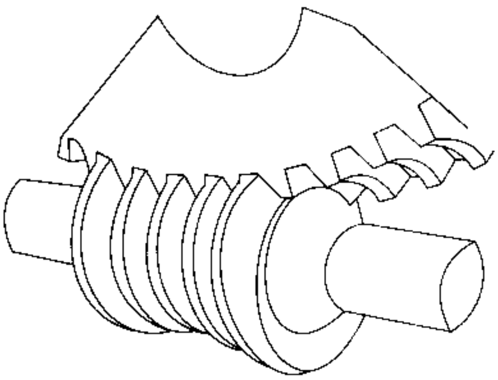
(d) Bevel gears



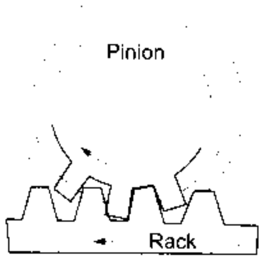
(e) Spiral gears



(f) Hypoid gears



(g) Worm and worm gear



(h) Rack and pinion

Fig.11.1 Types of gears

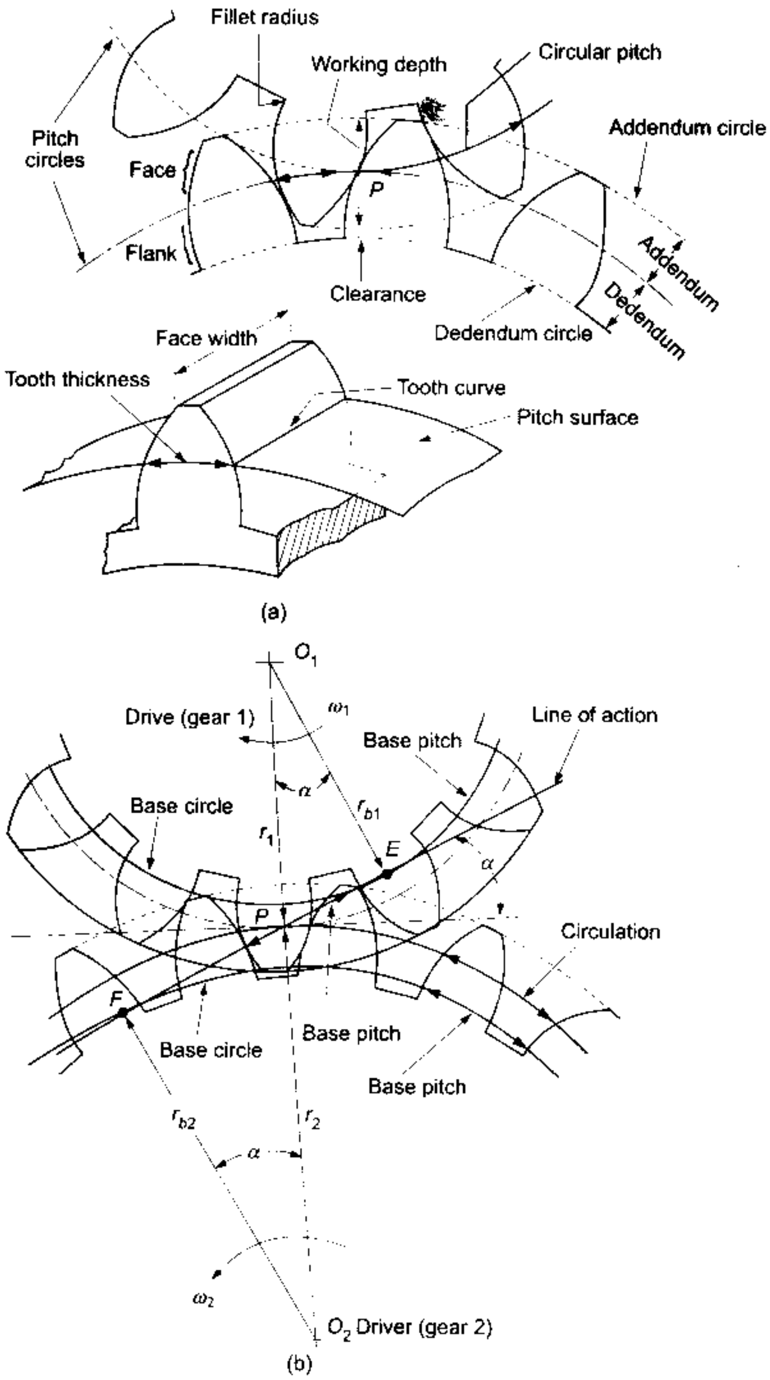


Fig.11.2 Gear nomenclature

Pitch point The pitch point is the pitch of the tangency or the point of contact of the two pitch circles of the mating gears.

Circular pitch (p) The circular pitch is the distance measured along the circumference of the pitch circle from a point on one tooth to a corresponding point on the adjacent tooth.

$$p = \frac{\pi d}{z} \tag{11.1}$$

where z is the number of teeth.

Base pitch (p_b) The base pitch is the distance measured along the circumference of the base circle from a point on one tooth to a corresponding point on the adjacent tooth.

Base pitch, $$p_b = p\cos\alpha \tag{11.2}$$

where α is the pressure angle of gear tooth profile.

Diametral pitch (P) The diametral pitch is expressed as the number of teeth per unit pitch circle diameter.

$$P = \frac{z}{d} \tag{11.3}$$

$$Pp = \pi \tag{11.4}$$

Module (m) Module is expressed as the length of the pitch circle diameter per unit number of teeth.

$$m = \frac{d}{z} = \frac{1}{P} \tag{11.5}$$

Addendum (h_a) Addendum is the radial height of the tooth above pitch circle.

Addendum circle The addendum circle is a circle bounding the top of the teeth.

Dedendum (h_f) Dedendum is the radial depth of a tooth below the pitch circle.

Dedendum circle The dedendum circle is a circle passing through the roots of all the teeth.

Clearance (c) Clearance is the radial height difference between addendum and dedendum of a teeth.

Face Face is the part of the tooth surface lying below the pitch surface.

Backlash Backlash is the minimum distance between the non driving side of a tooth and adjacent side of the mating tooth at the pitch circle.

Profile Profile is the curve forming face and flank.

Tooth thickness (t) Tooth thickness is the arc distance measured along the pitch circle from its intercept with one flank to its intercept with the other flank of the same tooth.

Face width (b) Face width is the width of the gear tooth measured axially along the pitch surface.

Top land Top land is the surface of the top of the tooth.

Tooth fillet Tooth fillet is the radius that connects the root circle to the profile of the tooth.

Tooth space Tooth space is the width of the space between two teeth measured on the pitch circle.

Pressure angle (α) The pressure angle is the angle between the common normal at the point of contact and the common tangent at the pitch point. The pressure angle is either 14.5° or 20°.

Path of contact The path of contact is the locus of the point of contact of two mating teeth from the beginning of engagement to the end of engagement. It is a straight line.

Path of approach The path of approach is the portion of the path of contact from the beginning of engagement to the pitch point.

Angle of approach The angle of approach is the angle turned by gears during path of approach.

Path of recess The path of recess is the portion of the path of contact from the pitch point to the end of engagement of the two mating teeth.

Angle of recess The angle of recess is the angle turned through during path of recess.

Arc of contact The arc of contact is the locus of a point on the pitch circle, from the beginning of engagement to the end of engagement of the pair of teeth in mesh.

Involute The involute of a circle is the curve traced by the end of a thread as it is unwound from a stationary cylinder.

Base Circle Diameter (d_b) The base circle diameter is the diameter of the base circle.

$$d_b = d\cos\alpha \tag{11.6}$$

Cycloid The cycloid is the locus of a point on the circumference of a circle which rolls without slipping on a fixed straight line.

Centre distance (C) The centre distance is the distance between the centres of rotation of the two gears in mesh.

$$C = \frac{d_1 + d_2}{2} = \frac{m\,(z_1 + z_2)}{2} \tag{11.7}$$

11.4 FUNDAMENTAL LAW OF GEARING

Let us consider two curved bodies 1 and 2 rotating about their centres O_1 and O_2 and contacting at point A, as shown in Fig.11.3. Here, A_1 and A_2 are two coincident points, A_1 lying on body 1 and A_2 lying on body 2. The common tangents and normals are TT' and NN', respectively at point A. Let ω_1 and ω_2 be the angular velocities of body 1 and 2 respectively. Let v_{a1} and v_{a2} be the linear velocities of point A. v_{a1} is perpendicular to O_1A_1 and v_{a2} is perpendicular to O_2A_2. Let the common normal intersect the line of centres at point P. Let O_1G and O_2H be perpendiculars to AP.

If the two bodies are to remain in contact, the component of velocities of A_1 and A_2 along the common normal must be equal.

Therefore
$$v_{a1}\cos\alpha = v_{a2}\cos\beta$$
or
$$\omega_1 \cdot O_1A_1\cos\alpha = \omega_2 \cdot O_2A_2\cos\beta$$
$$\omega_1 \cdot O_1A_1 \cdot \frac{A_1F}{A_1C} = \omega_2 \cdot O_2A_2 \cdot \frac{A_2F}{A_2B}$$

The condition for pure rolling is that the point of contact shall lie on the line of centres.

$$\frac{\omega_1}{\omega_2} = \frac{A_1C}{O_1A_1} \cdot \frac{O_2A_2}{A_2B}$$

Triangles A_1CF and O_1A_1G are similar. Also triangles A_2FB and O_2A_2H are similar.

Therefore
$$\frac{A_1C}{O_1A_1} = \frac{A_1F}{O_1G}$$
and
$$\frac{A_2B}{O_2A_2} = \frac{A_2F}{O_2H}$$
Hence
$$\frac{\omega_1}{\omega_2} = \frac{A_1F}{O_1G} \cdot \frac{O_2H}{A_2F}$$
But
$$A_1F = A_2F$$
$$\frac{\omega_1}{\omega_2} = \frac{O_2H}{O_1G}$$

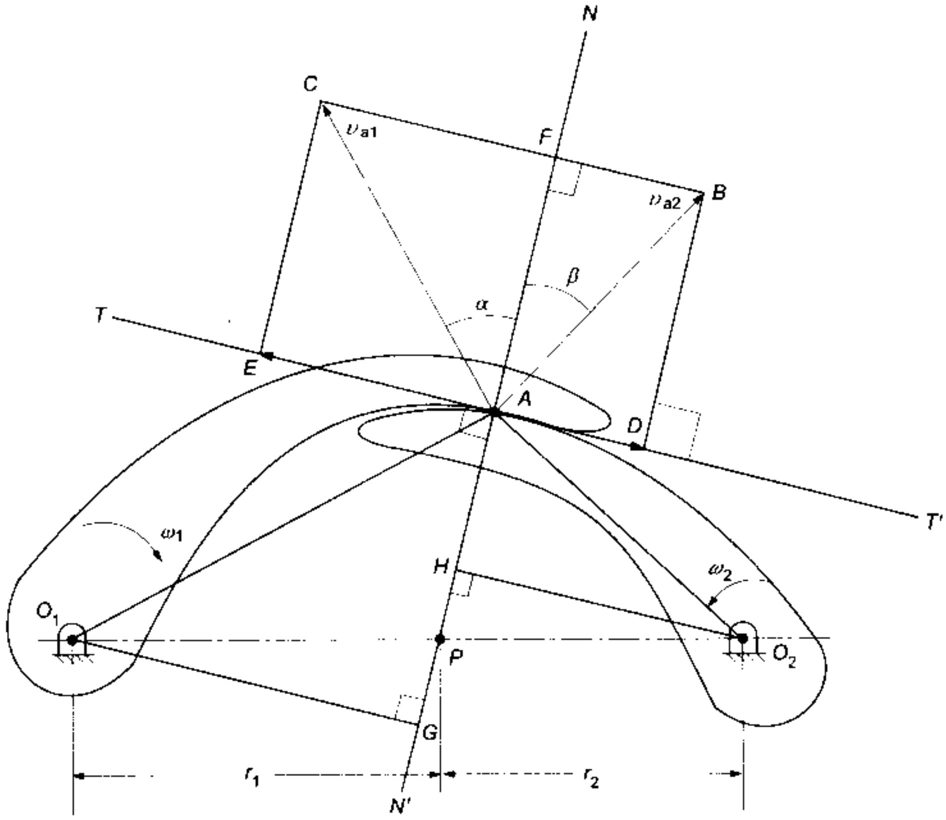


Fig.11.3 Law of gearing

Now triangles O_1PG and O_2PH are similar. Hence

$$\frac{O_2H}{O_1G} = \frac{O_2P}{O_1P}$$

or

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} \tag{11.8}$$

Therefore, for constant angular velocity ratio of the two gears in contact the common normal at the point of contact must always intersect the line of centres at a fixed point (pitch point) and divide this line in the inverse ratio of the angular velocities of the two gears. This is the fundamental law of gearing.

Conjugate action When the tooth profiles are so shaped so as to produce a constant angular velocity ratio during meshing, then the surfaces are said to be conjugate. Two conjugate surfaces in contact always satisfy the law of gearing.

11.5 RELATIVE VELOCITY BETWEEN GEAR TEETH

Considering Fig.11.3 again, the relative velocity along the tangent,

$$\begin{aligned} v_r &= v_{a1}\sin\alpha + v_{a2}\sin\beta \\ &= \omega_1 \cdot O_1A_1 \cdot \frac{FC}{A_1C} + \omega_2 \cdot O_2A_2 \cdot \frac{FB}{A_2B} \end{aligned}$$

Now $$\frac{A_1C}{O_1A_1} = \frac{A_1F}{O_1G} = \frac{FC}{AG}$$

and $$\frac{A_2B}{O_2A_2} = \frac{A_2F}{O_2H} = \frac{FB}{AH}$$

Hence $$\begin{aligned} v_r &= \omega_1 \cdot AG + \omega_2 \cdot AH \\ &= \omega_1 (AP + PG) + \omega_2 (AP - PH) \\ &= (\omega_1 + \omega_2)\, AP + \omega_1 \cdot PG - \omega_2 \cdot PH \end{aligned}$$

Now $$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PH}{PG} = \frac{r_2}{r_1}$$

Hence $$\begin{aligned} v_r &= (\omega_1 + \omega_2)\, AP \qquad (11.9) \\ &= \text{(Sum of the angular velocities)} \times \text{Distance of the point of contact from the pitch point.} \end{aligned}$$

11.6 EULER SAVARY EQUATION

The Euler Savary equation defines the relationship between two points, one in each body, when two bodies move relative to each other. These two points are related to each other in that each is the instantaneous centre of curvature of the path traced out by the other. In case of gears it connects the radii of curvature of the centrodes with the point of contact.

Consider two gears 1 and 2 rotating about their axis at O_1 and O_2, as shown in Fig.11.4. Let TT' and NN' be the common tangent and normal respectively. The point P, the pole (pitch point) must lie on the line of centers O_1O_2, by Kennedy's three centre theorem. Let the point of contact of the two gear teeth be at C. Then

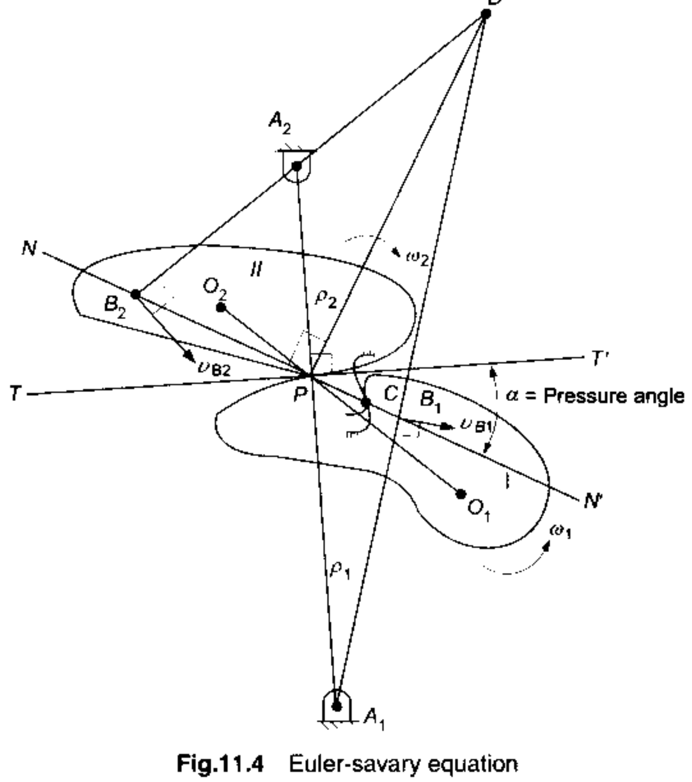


Fig.11.4 Euler-savary equation

$B_1C = \rho P_1$ = radius of curvature for the profile of gear teeth 1

$B_2C = \rho P_2$ = radius of curvature for the profile of gear teeth 2

Because point P cannot have velocity along A_1A_2 as A_1A_2 is a fixed link. Otherwise it will dig into or go apart. The only possibility is that it can have velocity along B_1B_2. Therefore, $PD \perp NN'$ and $A_1A_2 \perp TT'$.

Now $$\omega_1/\omega_2 = A_2P/A_1P = \rho_2/\rho_1$$

Join B_2A_2 and produce to meet PD at D. Also join DA_1, it will pass through B_1. Now, $PC = x$ = distance of contact point C from pole P. Velocity of B_2 is perpendicular to B_2A_2. Therefore, instantaneous centre of B_2 must lie on line B_2A_2 produced. Similarly, the instantaneous centre of B_1 must lie on A_1B_1 produced. They meet at D.

To find ρ_{P1} for a given ρ_1, ρ_2, ρ_{P2}, α and x, we proceed as follows:
Draw a horizontal line TT', as shown in Fig.11.5. Draw A_2PA_1 perpendicular to TT' and cut off $A_2P = \rho_2$ and $A_1P = \rho_1$. Draw NN' at an angle a with TT'. Cut off $PC = x$. Then cut off $B_2C = \rho_{P2}$. Join B_2A_2 and produce to meet PD at D. Join A_1D. Then $B_1C = \rho_{P1}$.

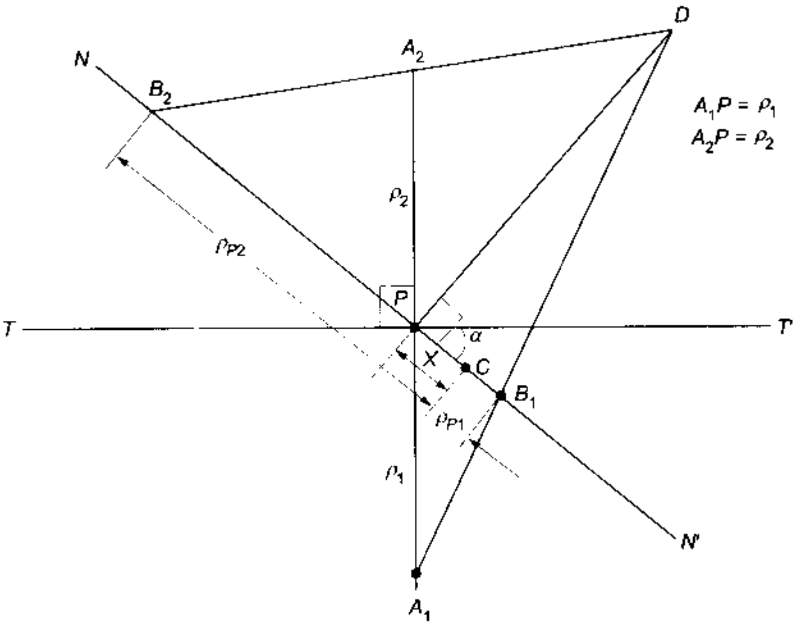


Fig.11.5 Euler-savary equation

The Euler-Savary equation can be written as:

$$\frac{1}{\rho_1} + \frac{1}{\rho_2} = \left[\frac{1}{(\rho_{P1} + x)} + \frac{1}{(\rho_{P2} - x)}\right] \sin\alpha \tag{11.10}$$

11.7 GEAR TOOTH FORMS

There are two types of gear tooth forms: involute and cycloidal. The involute of a circle is the curve traced by the end of a thread as it is unwound from a stationary cylinder. Cycloid is the locus of a point on the circumference of a circle which rolls without slipping on a fixed straight line. If the circle, instead of rolling without slipping on a straight line, rolls on the outside of another circle, the locus of the point on the circumference is called as epicycloid. Conversely, if the circle rolls on the outside of another circle, the corresponding locus of the point on the circumference of the rolling circle is called hypo-cycloid. We shall discuss the involute and cycloidal profiles in brief.

11.7.1 Cycloidal Tooth Profile

A cycloidal rack tooth profile meshing with a pinion is shown in Fig.11.6(a). The curve generated by a point on a circle 1 by its motion to the right on the straight line, which is the pitch line, is the profile of the face of the cycloidal tooth and the curve generated below the pitch line by a point on the rolling circle 2 is the flank of the tooth profile.

Fig.11.6(b) shows the construction of the cycloidal teeth profile of a gear. The circle 1 rolling on the outside of the pitch circle, generates a epicycloid, which is the face portion of the tooth profile. The circle 1 rolls without slipping to the right. The circle 2 rolls without slipping to the left on the inside of the circle generating a hypocycloid, representing the flank profile of the cycloidal tooth.

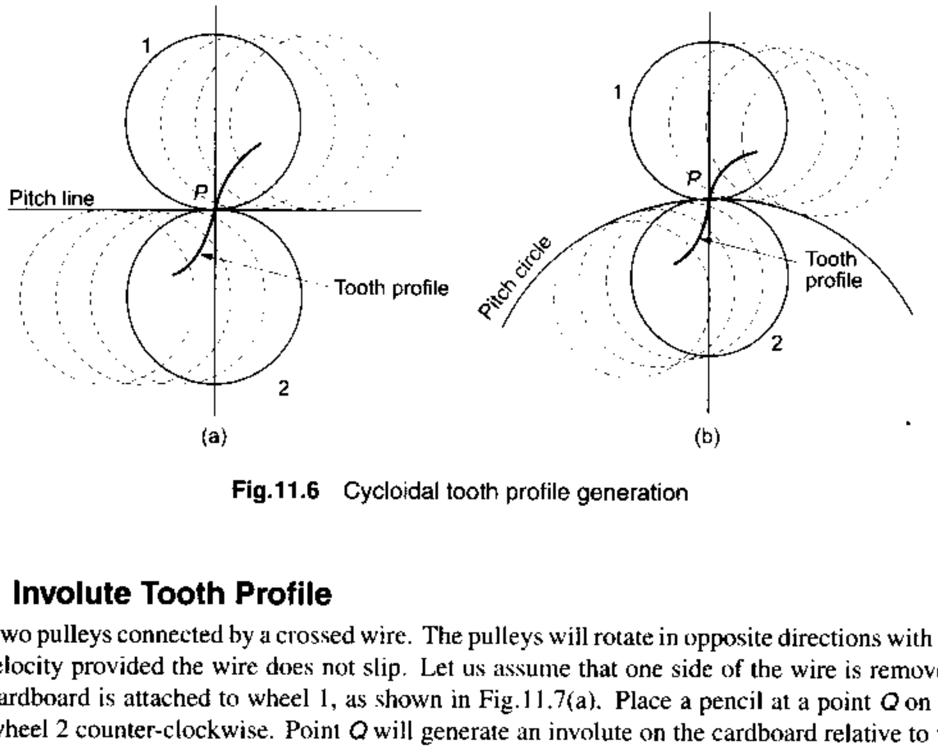


Fig.11.6 Cycloidal tooth profile generation

11.7.2 Involute Tooth Profile

Consider two pulleys connected by a crossed wire. The pulleys will rotate in opposite directions with constant angular velocity provided the wire does not slip. Let us assume that one side of the wire is removed and a piece of cardboard is attached to wheel 1, as shown in Fig.11.7(a). Place a pencil at a point Q on the wire and turn wheel 2 counter-clockwise. Point Q will generate an involute on the cardboard relative to wheel 1. If a cardboard is now attached to wheel 2, as shown in Fig.11.7(b), and the process is repeated, an involute is generated on the cardboard of wheel 2. If the cardboards are now cut along the involute, one side of tooth is formed on both the wheels.

The circles that have been used for generating the involutes are known as *base circles*. The angle that is included by a line perpendicular to the line of action through the centre of the base circle and a line from O_1 to O_2 through Q, is known as the *involute pressure angle*, as shown in Fig.11.7(c).

The intersection of the line perpendicular to the base circles and the line of centres has been labelled as P, the pitch point. The circles passing through point P with O_1 and O_2 as centres are called the *pitch circles*, as shown in Fig.11.7(d). At the pitch point, there is pure rolling, and at all other points there is a combination of rolling and sliding.

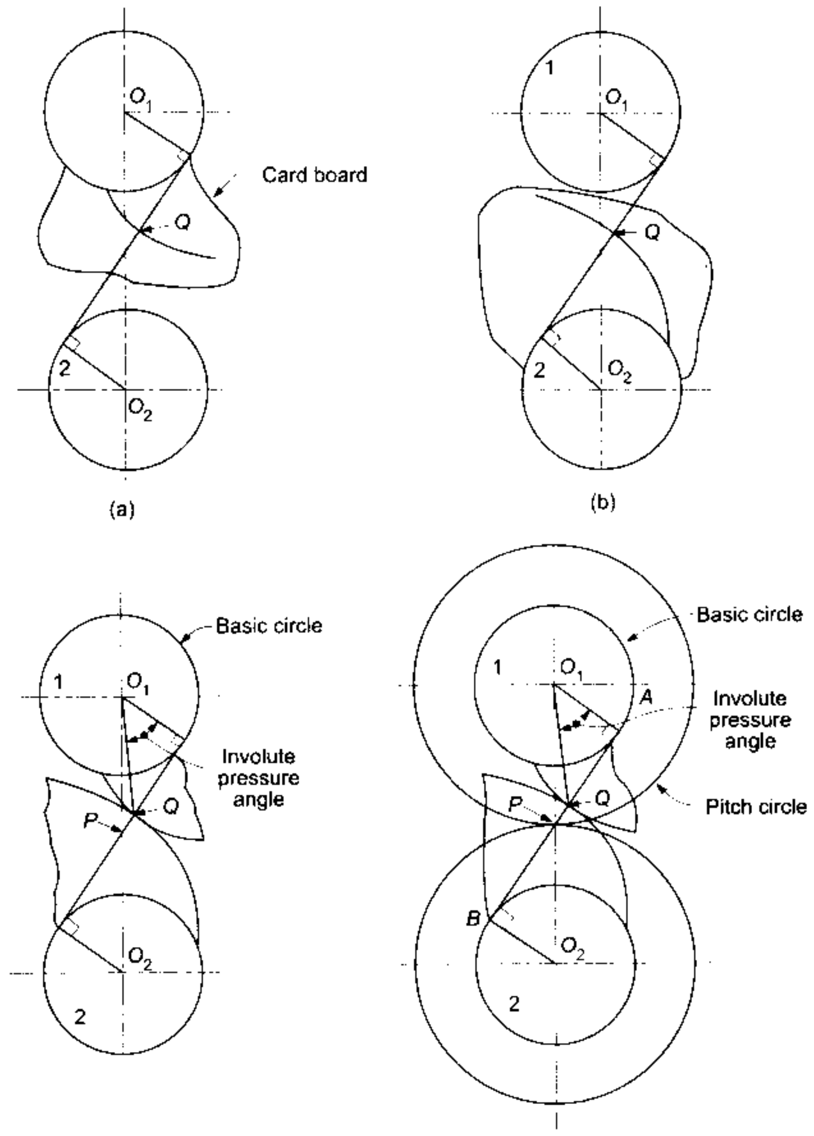


Fig.11.7 Involute tooth profile generation

11.7.3 Comparison between Involute and Cycloidal Tooth Profiles

The comparison of involute and cycloidal tooth profiles is given in Table 11.1. The cycloidal profile is not commonly used for gear tooth.

11.8 CONSTRUCTION OF AN INVOLUTE

The construction of the involute tooth profile is shown in Fig.11.8. The following steps may be followed to draw the involute:

1. Draw the base circle.

2. Divide the base circle quadrant into equal number of parts (say six). Mark the points *0* to *6* on the circumference of the circle.
3. Draw tangents at points *1* to *6*.
4. Cut off $1a = 01$ on tangent at *1*; $2b = 02$ on tangent at *2* and so on.
5. Join *0*, *a*, *b*, etc. by a smooth curve to obtain the involute profile.

Table 11.1 Comparison between involute and cycloidal tooth profiles.

Characteristic	Involute gears	Cycloidal gears
1. Pressure angle	Constant throughout the engagement	Varies from commencement to end
2. Ease of manufacture	Easy to manufacture	Difficult to manufacture
3. Centre distance	Do not require exact centre distance	Requires exact centre distance
4. Interference	May occur	No interference
5. Strength	Less	More
6. Wear	More	Less
7. Running	Smooth	Less smooth

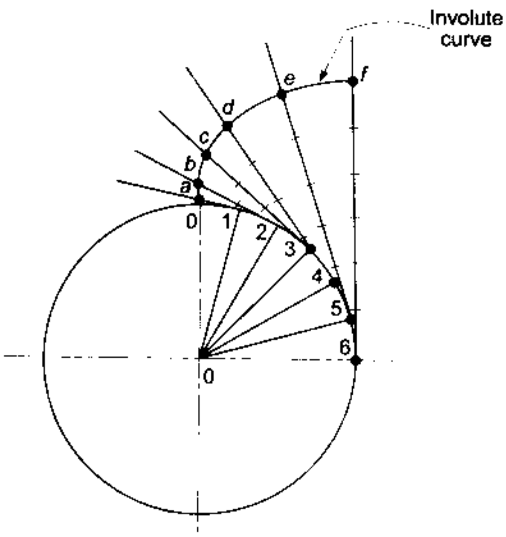


Fig.11.8 Involute profile

11.9 INVOLUTE FUNCTION

Consider the involute of a circle shown in Fig.11.9(a).

Let

$$l = \text{length of the thread unwrapped}$$
$$= r_b(\beta + \delta)$$
$$= r_b \tan \alpha$$

Thus

$$\beta + \delta = \tan \alpha$$

where r_b = base radius.

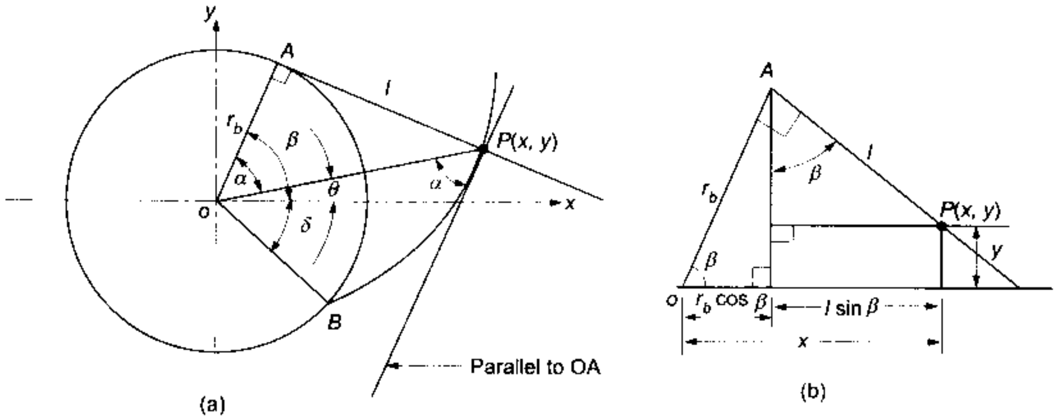


Fig.11.9 Involute function

Also from Fig.11.9(b), we have

$$x = r_b \cos\beta + l \sin\beta$$
$$= r_b[\cos\beta + (\beta+\delta)\sin\beta]$$

and
$$y = r_b[\sin\beta - (\beta+\delta)\cos\beta]$$

Also
$$\theta + \alpha = \beta = \tan\alpha - \delta$$

or
$$\theta = \tan\alpha - \alpha - \delta$$
$$= \text{inv}(\alpha) - \delta$$

where
$$\text{inv}(\alpha) = \tan\alpha - \alpha \quad (11.11)$$

The equation (11.11) represents the involute function. Its values are given in standard tables.

11.10 INVOLUTOMETRY

Figure 11.10 shows an involute which has been generated from a base circle of radius r_b. The involute contains two points *A* and *B* with corresponding radii r_A and r_B and involute pressure angles α_A and α_B.

$$r_b = r_A \cos\alpha_A$$

and
$$r_b = r_B \cos\alpha_B$$

Therefore
$$\cos\alpha_B = \left(\frac{r_A}{r_B}\right)\cos\alpha_A \quad (11.12)$$

It is possible to evaluate the involute pressure angle at any point on the involute profile from (11.12).

Now
$$\text{arc}DG = \text{length } BG$$
$$\angle DOG = \text{arc}\frac{DG}{OG} = \frac{BG}{OG}$$
$$\tan\alpha_B = \frac{BG}{OG}$$

Thus
$$\angle DOG = \tan\alpha_B$$

Also
$$\angle DOB = \angle DOG - \alpha_B$$
$$= \tan\alpha_B - \alpha_B$$

and $\angle DOA = \tan\alpha_A - \alpha_A$

Now $\angle DOE = \angle DOB + 0.5\dfrac{t_B}{r_B}$

$= \text{inv}(\alpha_B) + \dfrac{t_B}{2r_B}$

Also $\angle DOE = \angle DOA + 0.5\dfrac{t_A}{r_A}$

$= \text{inv}(\alpha_A) + \dfrac{t_A}{2r_A}$

Thus
$$t_B = 2r_B\left[\frac{t_A}{2r_A} + inv(\alpha_A) - \text{inv}(\alpha_B)\right] \qquad (11.13)$$

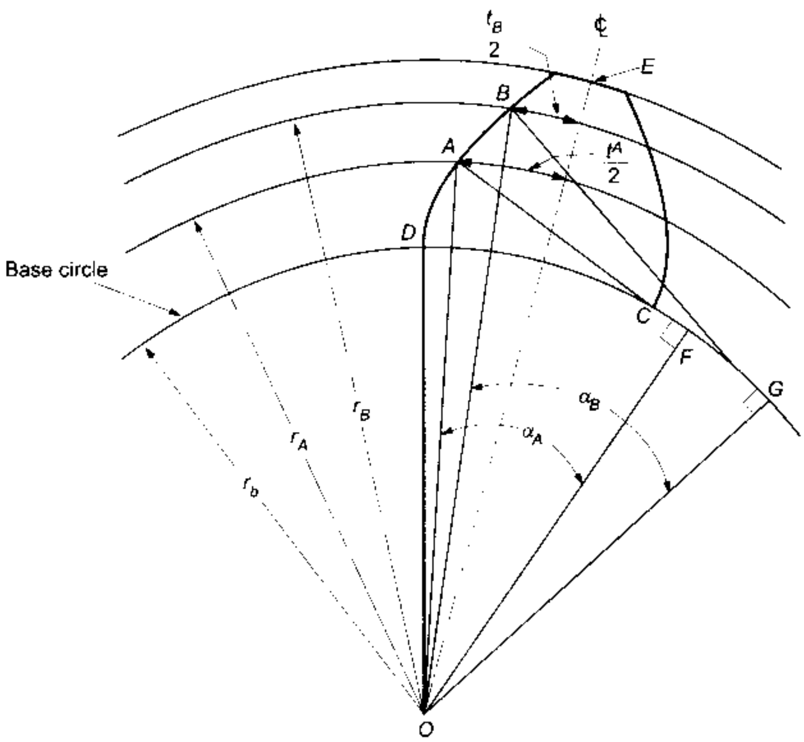


Fig.11.10 Involutometry

11.11 INVOLUTE GEAR TOOTH ACTION

The gear tooth action between two gears is shown in Fig.11.11. *P* is the pitch point and line *EF* is tangent to both the base circles, along which all points of contact of the two teeth must lie. Line *EF* is called the line of action or the pressure line. Line *XX'* is perpendicular to the line of centres at the pitch point. The angle between *XX'* and *EF* is called the pressure angle. If one gear rotates in clockwise direction then the other gear would rotate in the reverse direction of counter-clockwise.

The teeth first comes into contact at point *A*, where the addendum circle of the driven gear cuts the line of action. Contact follows the line of action through point *P*, and contact ceases at point *B*, where the addendum circle of the driving gear cuts the line of action. Line *AB* is called the path of the point of contact, and its length is the length of the path of contact. Point C is the intersection of the tooth profile on gear 2 with its pitch circle when the tooth is at the beginning of contact, and point G is the same point on the profile when the tooth is at the end of contact. Points D and H are the corresponding for gear 1. Arcs CPG and DPH are the arcs on the pitch circles through which the mating tooth profiles move as they pass from the initial to the final point of contact. These arcs are known as the *arcs of action*.

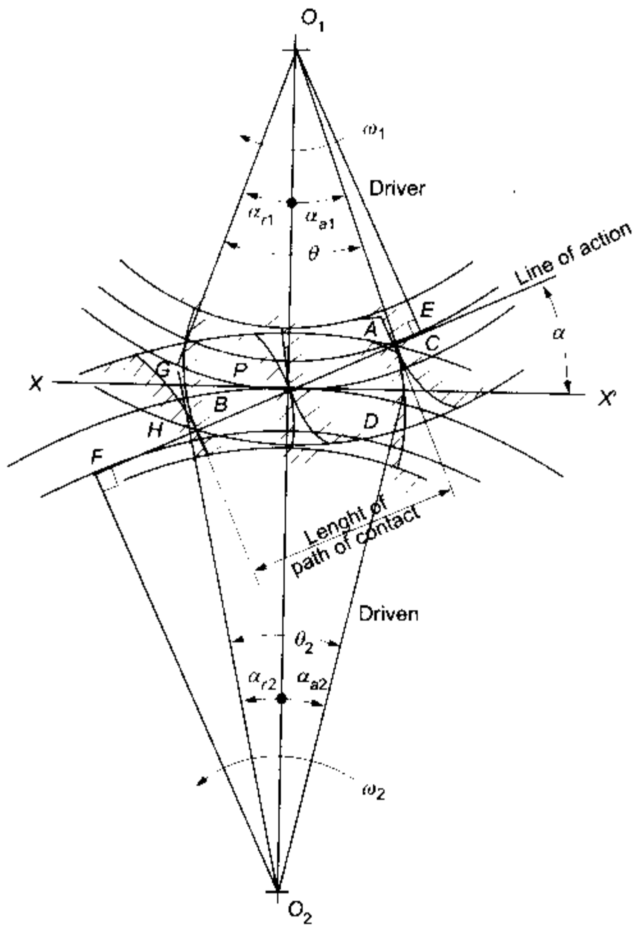


Fig.11.11 Involute gear tooth action

Since the pitch circles roll on one another, these are equal. The angles θ_1 and θ_2 which subtend these arcs are called the *angles of action*. The angles of action are divided into two parts called the *angles of approach* (α_a) and *angles of recess* (α_r). The angle of approach is defined as the angle through which a gear rotates from the instant a pair of teeth comes into contact until the teeth are in contact at the pitch point. The angle of recess is the angle through which a gear rotates from the instant the teeth are in contact at the pitch point until contact is broken. In general, the angle of approach is not equal to the angle of recess. Gear tooth action is smoother in recess than in approach.

From Fig.11.12, we have

$$\alpha_{r1} = \sin^{-1}\left(\frac{PO_1 \sin\alpha_a}{AO_1}\right)$$

$$\theta_1 = 180° - (\alpha_a + \alpha_{r1})$$

$$AP = \frac{AO_1 \sin\theta_1}{\sin\alpha_a}$$

$$\alpha_{r2} = \sin^{-1}\left(\frac{PO_2 \sin\alpha_a}{BO_2}\right)$$

$$\theta_2 = 180° - (\alpha_a + \alpha_{r2})$$

$$BP = \frac{BO_2 \sin\theta_2}{\sin\alpha_a}$$

$$\alpha_{a1} = \frac{AP}{r_1} \quad (11.14)$$

$$\alpha_{r1} = \frac{BP}{r_1} \quad (11.15)$$

$$\alpha_{a2} = \frac{AP}{r_2} \quad (11.16)$$

$$\alpha_{r2} = \frac{BP}{r_2} \quad (11.17)$$

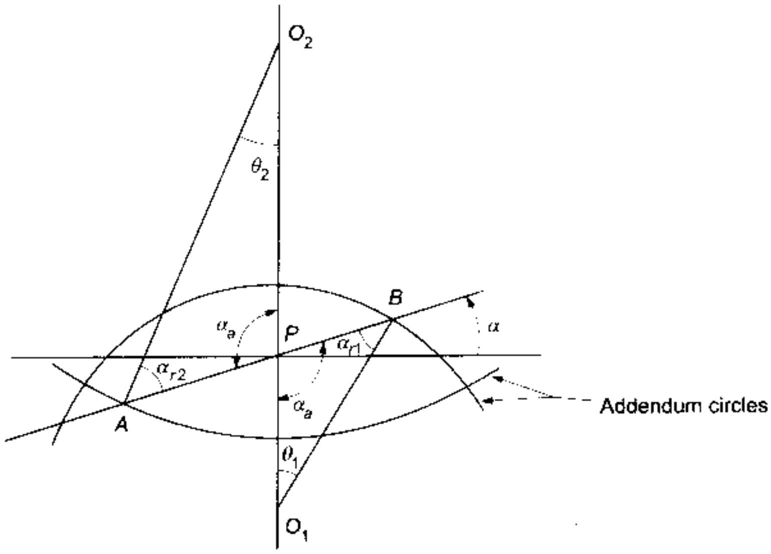


Fig.11.12 Angles of action

11.12 CHARACTERISTICS OF INVOLUTE ACTION

The three characteristics of the involute action are arc of contact, length of path of contact and the contact ratio. As shown in Fig.11.13, the contact of two gear teeth begins at A and ends at B.

Addendum radius of pinion, $r_{a1} = r_1 + h_{a1}$
Base circle radius of pinion, $r_{b1} = r_1 \cos\alpha$
Addendum radius of gear, $r_{a2} = r_2 + h_{a2}$
Base circle radius of gear, $r_{b2} = r_2 \cos\alpha$

where r_1 = pitch circle radius of pinion
r_2 = pitch circle radius of gear

h_{a1} = addendum of pinion
h_{a2} = addendum of gear
r_{b1} = base circle radius of pinion
r_{b2} = base circle radius of gear

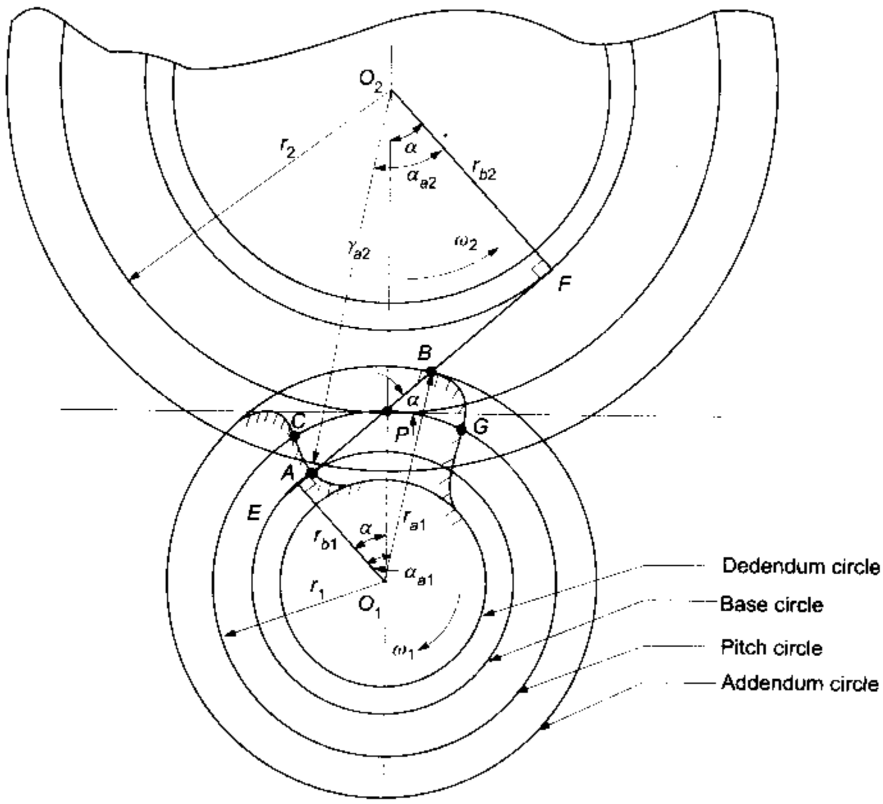


Fig.11.13 Angles of action

Length of path of recess, $L_r = PB = EB - EP$

$$= \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} - O_1 P \sin\alpha$$

$$= \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} - r_1 \sin\alpha$$

Length of path of approach, $L_a = AP = AF - PF$

$$= \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - O_2 P \sin\alpha$$

$$= \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - r_2 \sin\alpha$$

Length of path of contact, $AB = L_p = L_r + L_a$

$$= \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} + \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - (r_1 + r_2)\sin\alpha \quad (11.18)$$

Length of arc of contact, $L_c = \text{arc}\,CG$

$$= \frac{AB}{\cos\alpha} \quad (11.19)$$

Maximum length of path of recess $= r_2 \sin\alpha$

Maximum length of path of approach $= r_1 \sin\alpha$

The *contact ratio* is defined as the average number of pairs of teeth which are in contact. This can be found by noting how many times the base pitch fits into the length of the path of contact. The contact ratio (m_c) can be expressed as:

$$m_c = \text{length of path of contact / base pitch} = \frac{AB}{p_b}$$

$$= \frac{L_p}{p_b} \quad (11.20)$$

where $p_b = p\cos\alpha = \pi m \cos\alpha$

For a rack and a pinion,

$$L_p = \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} - r_1 \sin\alpha + \frac{a}{\sin\alpha} \quad (11.21)$$

where a = addendum.

Example 11.1

A pinion of 24 teeth drives a gear of 60 teeth at a pressure angle of 20°. The pitch radius of the pinion is 38 mm and the outside radius is 41 mm. The pitch radius of the gear is 95 mm and the outside radius is 98.5 mm. Calculate the length of action and contact ratio.

■ Solution

Length of path of contact,

$$L_p = \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} + \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - (r_1 + r_2)\sin\alpha$$

Here, $r_{a1} = 41$ mm, $r_{a2} = 98.5$ mm, $r_1 = 38$ mm, $r_2 = 95$ mm, $\alpha = 20°$.

$$r_{b1} = r_1 \cos\alpha = 38\cos 20° = 35.7 \text{ mm}$$

$$r_{b2} = r_2 \cos\alpha = 95\cos 20° = 811.27 \text{ mm}$$

$$L_p = \left(41^2 - 35.7^2\right)^{0.5} + \left(98.5^2 - 811.27^2\right)^{0.5} - (38 + 95)\sin 20°$$

$$= 20.16 + 41.63 - 45.49$$

$$= 16.30 \text{ mm}$$

Contact ratio, $m_c = \dfrac{L_p}{p_b}$

$$p_b = 2\pi \frac{r_{b1}}{z_1}$$

$$= 2\pi \times \frac{35.7}{24} = 11.34 \text{ mm}$$

$$m_c = \frac{16.30}{11.34} = 1.744$$

Example 11.2

Two equal size spur gears in mesh have 36 number of teeth, 20° pressure angle and 6 mm module. If the arc of contact is 1.8 times the circular pitch, find the addendum.

■ **Solution**

Circular pitch, $p = \pi m = \pi \times 6 = 18.85$ mm

Length of arc of contact, $L_a = 1.8p = 33.93$ mm

Length of path of contact, $L_p = L_a \cos\alpha$

$= 33.93 \cos 20° = 31.88$ mm

Pitch radii, $r_1 = r_2 = \dfrac{mz}{2} = 6 \times \dfrac{36}{2} = 108$ mm

$$L_p = \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} + \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - (r_1 + r_2)\sin\alpha$$

Here $r_{a1} = r_{a2}$ and $r_{b1} = r_{b2} = r\cos\alpha = 108\cos 20° = 101.49$ mm

$$L_p = 2\left(r_a^2 - 101.49^2\right)^{0.5} - 216\sin 20°$$

$$31.88 = 2\left(r_a^2 - 101.49^2\right)^{0.5} - 73.87$$

$$r_a = 114.44 \text{ mm}$$

Addendum, $h_a = r_a - r = 114.44 - 108 = 6.44$ mm

Example 11.3

Two 20° involute gears in mesh have a gear ratio of 2 and 20 teeth on the pinion. The module is 5 mm and the pitch line speed is 1.5 m/s. Assuming addendum to be equal to one module, find (a) angle turned through by pinion when one pair of teeth is in mesh, and (b) maximum velocity of sliding.

■ **Solution**

(a)

$$r_1 = \frac{mz_1}{2} = 5 \times \frac{20}{2} = 50 \text{ mm}$$

$$r_2 = ir_1 = 2r_1 = 100 \text{ mm}$$

$$r_{a1} = r_1 + h_{a1} = 50 + 5 = 55 \text{ mm}$$

$$r_{a2} = r_2 + h_{a2} = 100 + 5 = 105 \text{ mm}$$

$$r_{b1} = r_1\cos 20° = 50\cos 20° = 46.98 \text{ mm}$$

$$r_{b2} = r_2\cos 20° = 100\cos 20° = 93.97 \text{ mm}$$

Length of path of approach, $L_a = \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2\sin\alpha$

$= \left[105^2 - 93.97^2\right]^{0.5} - 100\sin 20°$

$= 12.65$ mm

Length of path of recess, $L_r = \left[r_{a1}^2 - r_{b1}^2\right]^{0.5} - r_1\sin 20°$

$= \left[55^2 - 46.98^2\right]^{0.5} - 50\sin 20°$

$= 11.50$ mm

Length of path of contact, $L_p = L_a + L_r = 12.65 + 11.50 = 24.15$ mm

Length of arc of contact, $L_c = \dfrac{L_p}{\cos\alpha} = \dfrac{24.15}{\cos 20°} = 25.7$ mm

$$\text{Angle turned through by the pinion} = L_c \times \frac{360}{2\pi r_1}$$
$$= 25.7 \times \frac{360}{2\pi \times 50}$$
$$= 211.45°$$

(b) Maximum velocity of sliding, $v_s = (\omega_1 + \omega_2)\, L_a$

$$\omega_1 = \frac{v}{r_1} = \frac{1.5}{0.05} = 30 \text{ rad/s}$$
$$\omega_2 = \frac{v}{r_2} = \frac{1.5}{0.1} = 15 \text{ rad/s}$$
$$v_s = (30 + 15)12.65 = 5611.25 \text{ mm/s}$$

Example 11.4

The pressure angle of two gears in mesh is 20° and have a module of 10 mm. The number of teeth on pinion are 24 and on gear 60. The addendum of pinion and gear is same and equal to one module. Determine (a) the number of pairs of teeth in contact, (b) the angle of action of pinion and gear, and (c) the ratio of sliding to rolling velocity at the beginning of contact, at pitch point and at the end of contact.

■ **Solution**

$$\alpha = 20°, m = 10 \text{ mm}, z_1 = 24, z_2 = 60, h_{a1} = h_{a2} = 1 \text{ m} = 10 \text{ mm}$$
$$r_1 = \frac{mz_1}{2} = \frac{10 \times 24}{2} = 120 \text{ mm}, r_2 = \frac{10 \times 60}{2} = 300 \text{ mm}$$
$$r_{a1} = r_1 + h_{a1} = 120 + 10 = 130 \text{ mm}$$
$$r_{a2} = r_2 + h_{a2} = 300 + 10 = 310 \text{ mm}$$
$$r_{b1} = r_1 \cos a = 120 \times \cos 20° = 112.76 \text{ mm}$$
$$r_{b2} = r_2 \cos a = 300 \times \cos 20° = 281.91 \text{ mm}$$

Length of path of recess, $L_r = \left[r_{a1}^2 - r_{b1}^2\right]^{0.5} - r_1 \sin\alpha$

$$= \left[130^2 - (112.76)^2\right]^{0.5} - 120 \sin 20°$$
$$= 23.65 \text{ mm}$$

Length of path of approach, $L_a = \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2 \sin\alpha$

$$= \left[310^2 - (281.91)^2\right]^{0.5} - 300 \cos 20°$$
$$= 26.33 \text{ mm}$$

Length of path of contact, $L_p = L_a + L_r = 26.33 + 23.65 = 411.98$ mm

$$\text{(a) Number of pairs of teeth in contact} = \frac{L_p}{\pi m \cos\alpha} = \frac{411.98}{\pi \times 10 \times \cos 20°}$$
$$= 1.69$$

(b) Angle of action of pinion, $\alpha_{a1} = \text{Arc of contact} \times \dfrac{360}{2\pi r_1}$

$$= \left(\frac{L_p}{\cos\alpha}\right) \times \frac{360}{2\pi r_1}$$

$$= \left(\frac{411.98}{\cos 20°}\right) \times \frac{360}{2\pi \times 120}$$

$$= 25.4°$$

Angle of action of gear, $\alpha_{a2} = \text{Arc of contact} \times \frac{360}{2\pi r_2}$

$$= \left(\frac{L_p}{\cos\alpha}\right) \times \frac{360}{2\pi r_2}$$

$$= \left(\frac{411.98}{\cos 20°}\right) \times \frac{360}{2\pi \times 300}$$

$$= 10.16°$$

(c) Ratio of sliding to rolling velocity $= \frac{v_s}{v_r}$

$$v_r = r_1\omega_1 = 120\omega_1$$

$$\omega_2 = \frac{24\omega_1}{60} = 0.4\omega_1$$

At the beginning of contact $= \dfrac{\omega_1 + \omega_2}{\dfrac{L_a}{v_r}}$

$$= \frac{(\omega_1 + 0.4\omega_1) \times 26.33}{120\omega_1}$$

$$= 0.3072$$

At the pitch point, $v_s = 0.$ Hence $\frac{v_s}{v_r} = 0$

At the end of contact $= \dfrac{\omega_1 + \omega_2}{\dfrac{L_r}{v_r}}$

$$= \frac{(\omega_1 + 0.4\omega_1) \times 23.65}{120\omega_1}$$

$$= 0.276$$

Example 11.5

Two 15 mm module, 20° pressure angle spur gears have addendum equal to one module. The pinion has 25 teeth and the gear 50 teeth. Determine whether interference will occur or not. If it occurs, to what value should the pressure angle be changed to eliminate interference.

■ **Solution**

$$\alpha = 20°, m = 15 \text{ mm}, z_1 = 25, z_2 = 50, h_{a1} = h_{a2} = 15 \text{ and } m = 10 \text{ mm}$$

Let the pinion be the driver.

$$r_1 = \frac{mz_1}{2} = \frac{15 \times 25}{2} = 187.5 \text{ mm}, r_2 = \frac{15 \times 50}{2} = 375 \text{ mm}$$

$$r_{a1} = r_1 + h_{a1} = 187.5 + 15 = 202.5 \text{ mm}$$

$$r_{a2} = r_2 + h_{a2} = 375 + 15 = 390 \text{ mm}$$

$$r_{b2} = r_2 \cos\alpha = 375 \times \cos 20° = 352.4 \text{ mm}$$

Maximum permissible length of path of approach,

$$(L_a)_{max} = r_1 \sin\alpha$$
$$= 187.5 \sin 20°$$
$$= 64.13 \text{ mm}$$

Length of path of approach,

$$L_a = \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2 \sin\alpha$$
$$= \left[390^2 - (352.4)^2\right]^{0.5} - 375 \sin 20°$$
$$= 38.81 \text{ mm}$$

Since $L_a < (L_a)_{max}$, hence interference will occur.
For $L_a = (L_a)_{max}$, we have

$$r_1 \sin\alpha = \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2 \sin\alpha$$
$$64.13 = \left[390^2 - (375 \cos\alpha)^2\right]^{0.5} - 375 \sin\alpha$$
$$(64.13 + 375 \sin\alpha)^2 = 390^2 - (375\cos\alpha)^2$$
$$4112.65 + 140625 + 48097.5 \sin\alpha = 152100$$
$$\sin\alpha = 0.15307$$
$$\alpha = 8.8°$$

Example 11.6

For a pair of involute spur gears, $m = 10$ mm, $\alpha = 20°$, $z_1 = 20$, $z_2 = 40$, $n_1 = 60$ rpm. The addendum on each gear is such that the path of approach and the path of recess on each side is 50% of the maximum possible length. Determine the addendum for the pinion and the gear and the length of arc of contact.

■ **Solution**

$$r_1 = mz_1/2 = 10 \times 20/2 = 100 \text{ mm}, r_2 = mz_2/2 = 10 \times 40/2 = 200 \text{ mm}$$

Let the pinion be the driver.

$$\text{Maximum possible length of approach} = r_1 \sin\alpha = 100 \sin 20° = 34.2 \text{ mm}$$
$$\text{Actual length of approach} = \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2 \sin\alpha$$
$$= \left[r_{a2}^2 - (200\cos 20°)^2\right]^{0.5} - 200 \sin 20°$$
$$= \left[r_{a2}^2 - 35321\right]^{0.5} - 68.4 = 0.5 \times 34.2 = 17.1$$

or

$$r_{a2}^2 - 35321 = (85.5)^2$$
$$r_{a2} = 206.5 \text{ mm}$$
$$h_{a2} = 206.5 - 200 = 6.05 \text{ mm}$$
$$\text{Maximum possible length of recess} = r_2 \sin\alpha$$
$$= 200 \sin 20° = 68.4 \text{ mm}$$
$$\text{Actual length of recess} = \left[r_{a1}^2 - r_{b1}^2\right]^{0.5} - r_1 \sin\alpha$$
$$= \left[r_{a1}^2 - (100\cos 20°)^2\right]^{0.5} - 100 \sin 20°$$

$$= \left[r_{a1}^2 - 8830\right]^{0.5} - 34.2 = 0.5 \times 68.4 = 34.2$$

or $$r_{a1}^2 - 8830 = (68.4)^2$$

$$r_{a1} = 116.2 \text{ mm}$$

$$h_{a1} = 116.2 - 100 = 16.2 \text{ mm}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\alpha} = 0.5\,(r_1 + r_2)\frac{\sin\alpha}{\cos\alpha}$$

$$= 0.5(100 + 200)\tan 20°$$

$$= 54.6 \text{ mm}$$

Example 11.7

Two involute gear wheels having module 3 mm and pressure angle 20° mesh externally to give a velocity ratio of 3. The pinion rotates at 75 rpm and addendum is equal to one module. Determine (a) the number of teeth on each wheel so that interference is just avoided, (b) the length of path and arc of contact, (c) the number of pairs of teeth in contact, and (d) the maximum velocity of sliding between the teeth.

■ **Solution**

$$m = 3 \text{ mm}, \alpha = 20°, i = 3, n_1 = 75 \text{ rpm}$$

$$r_{a1} = r_1 + h_{a1} = r_1 + 3, r_{a2} = r_2 + h_{a2} = 3r_1 + 3 \qquad [\because \quad r_2 = 3r_1]$$

$$r_{b1} = r_1\cos 20° = 0.9397r_1, r_{b2} = 3r_1\cos 20° = 2.819r_1$$

(a) Let the pinion be the driver.

$$L_a = \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2\sin\alpha$$

$$(L_a)_{max} = r_1\sin\alpha$$

To avoid interference, $$L_a = (L_a)_{max}$$

$$\left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - r_2\sin\alpha = r_1\sin\alpha$$

$$r_{a2}^2 - r_{b2}^2 = (r_1 + r_2)^2\sin^2\alpha$$

$$(3r_1 + 3)^2 - (2.819)^2 r_1^2 = (4r_1)^2\sin^2\alpha$$

or $$0.8186r_1^2 - 18r_1 - 9 = 0$$

$$r_1 = 22.48 \text{ mm}$$

$$r_2 = 67.44 \text{ mm}$$

$$z_1 = 2r_1/m = 2 \times 22.48/3 = 14.98 \cong 15,$$

so that $r_1 = 22.5$ mm, and $z_2 = 45$

(b) $$r_{a1} = 22.5 + 3 = 25.5 \text{ mm}, r_{a2} = 67.5 + 3 = 70.5 \text{ mm}$$

$$r_{b1} = 22.5\cos 20° = 21.143 \text{ mm}, r_{b2} = 67.5\cos 20° = 63.429 \text{ mm}$$

$$L_p = \left(r_{a1}^2 - r_{b1}^2\right)^{0.5} + \left(r_{a2}^2 - r_{b2}^2\right)^{0.5} - (r_1 + r_2)\sin\alpha$$

$$= \left[(25.5)^2 - (21.143)^2\right]^{0.5} + \left[(70.5)^2 - (63.429)^2\right]^{0.5}$$

$$-(22.5 + 67.5)\sin 20°$$

$$= 14.247 \text{ mm}$$

$$p = \pi m = \pi \times 3 = 11.425 \text{ mm}$$

Length of arc of contact. $L_c = \dfrac{14.247}{\cos 20°} = 15.16$ mm

(c) Number of pairs of teeth in contact $= \dfrac{L_c}{p} = \dfrac{15.16}{11.425} = 1.6$

(d) Maximum velocity of sliding $= (\omega_1 + \omega_2) r_1 \sin\alpha$

$$= \left(\frac{2\pi}{60}\right)(75 + 25) \times 22.5 \times \sin 20°$$

$$= 80.58 \text{ mm/s}$$

11.13 UNDERCUTTING IN INVOLUTE GEARS

An involute starts at the base circle and is generated outwards. It is therefore impossible to have an involute inside the base circle. The line of action is tangent to the two base circles of a pair of gears in mesh, and these two points represent the extreme limits of the length of action. These two points are called *interference points*. If the teeth are of such proportion that the beginning of contact occurs before the interference point is met, then the involute portion of the driven gear will mate with a noninvolute portion of the driving gear, and involute interference is said to occur. This condition is shown in Fig.11.14; E_1 and E_2 show the interference points that should limit the length of action. *A* shows the beginning of contact, and *B* shows the end of contact. It can be seen that the beginning of contact occurs before the interference point E_1 is met; therefore, interference is present. The tip of the driven tooth will gouge out or undercut the flank of the driving tooth as shown by the dotted line.

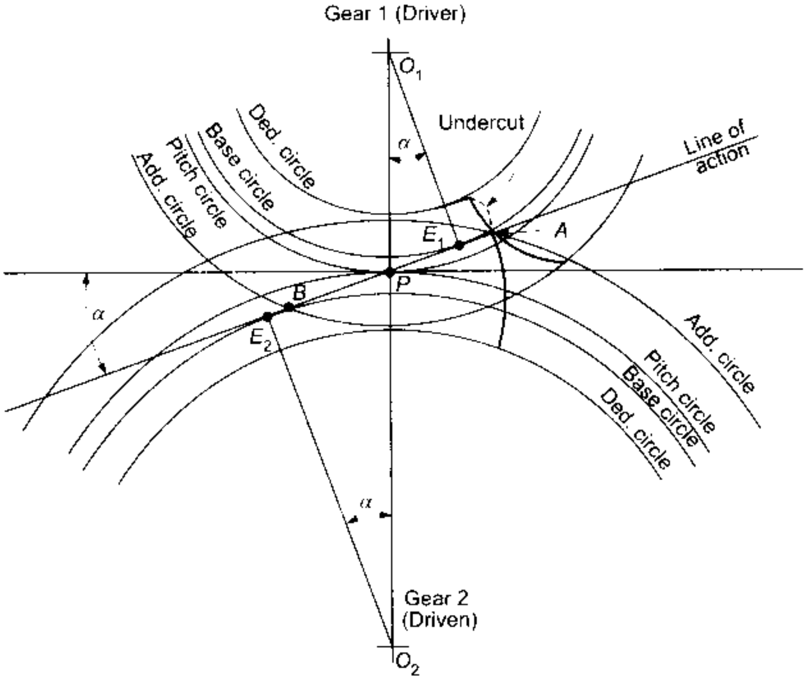


Fig.11.14 Interference in rack and pinion

There are several ways of eliminating interference. Interference can be avoided by undercutting, making stub teeth, increasing the pressure angle, and cutting the gears with long and short addendum gear teeth. The method of undercutting is to limit addendum of the driven gear so that it passes through the interference point E_1, thus giving a new beginning of contact. Interference and the resulting undercutting not only weaken the pinion tooth but may also remove a small portion of the involute adjacent to the base circle, which may cause a serious reduction in the length of action.

Fig.11.15 shows a rack and a pinion in mesh. The point of tangency of the line of action and the base circle of the pinion is labelled as the interference point *E*, which fixes the maximum addendum for the rack. The contact begins at *A*, and undercutting will occur as shown by the dotted line. If the addendum of the rack extends only to the line that passes through the interference point *E*, then the interference point becomes the beginning of contact, and interference is eliminated. If the number of teeth on the pinion is such that it will mesh without interference, it will mesh without interference with any other gear having the same or a larger number of teeth.

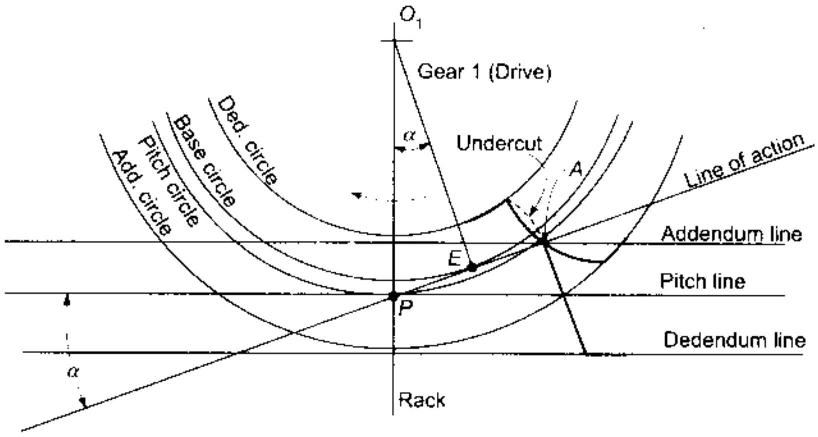


Fig.11.15 Interference in rack and pinion

11.14 MINIMUM NUMBER OF TEETH

11.14.1 Gear Wheel

For minimum number of teeth to avoid interference, the common tangent to the base circles cuts the addenda circles at *A* and *B*, as shown in Fig.11.16.

Let speed ratio, $i = \dfrac{z_2}{z_1}$

Addendum of pinion, $h_{a1} = a_p m$

Addendum of gear wheel, $h_{a2} = a_w m$

where a_p and a_w are the constants by which the module must be multiplied to get the addendum of pinion and gear wheel respectively.

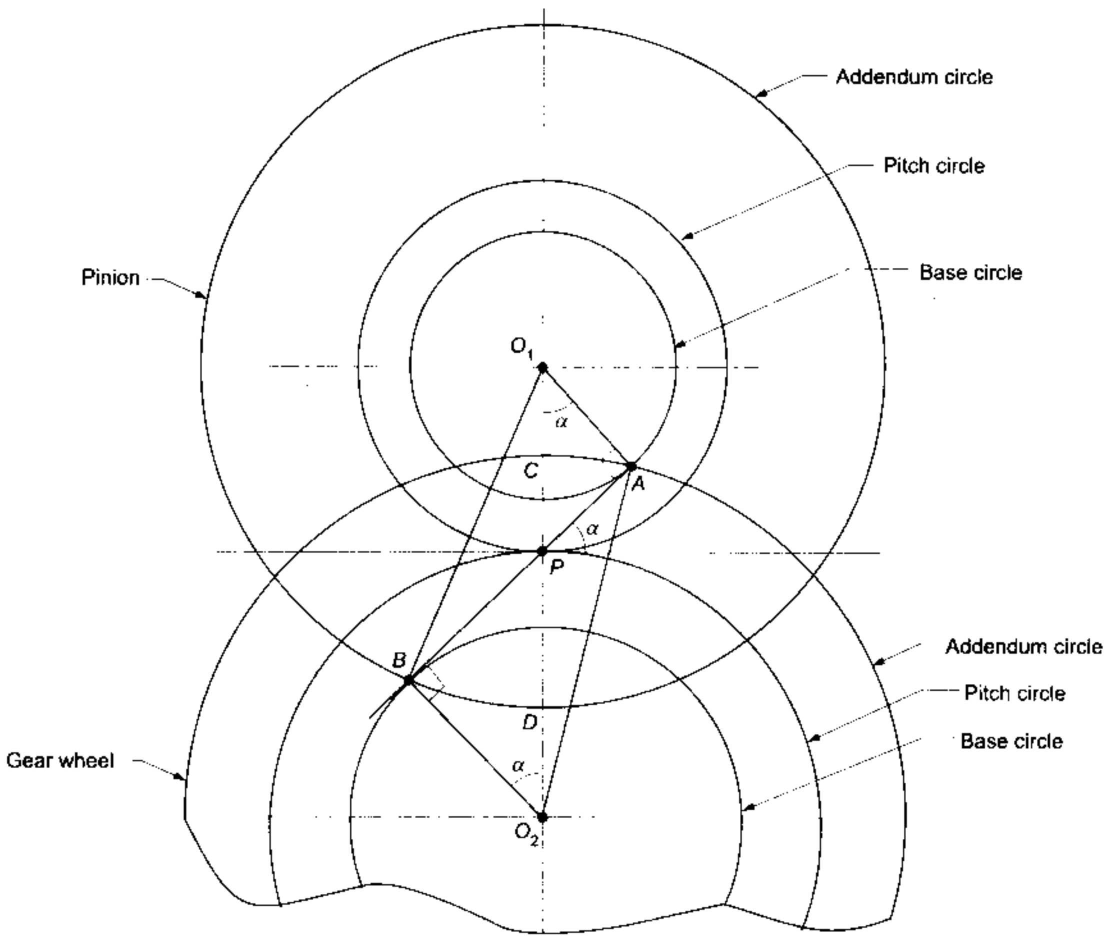


Fig.11.16 Calculating minimum number of teeth on pinion and gear wheel

From ΔAO_2P, we have

$$O_2A^2 = O_2P^2 + AP^2 - 2 \cdot O_2P \cdot AP \cdot \cos \angle O_2PA$$

$$= \left(\frac{mz_2}{2}\right)^2 + (O_1P\sin\alpha)^2 - 2 \cdot \frac{mz_2}{2} \cdot O_1P\sin\alpha \cdot \cos(90^\circ + \alpha)$$

$$= \left(\frac{mz_2}{2}\right)^2 + \left(\frac{mz_1}{2}\right)^2 \sin^2\alpha + 2 \cdot \left(\frac{mz_2}{2}\right) \cdot \left(\frac{mz_1}{2}\right) \sin^2\alpha$$

$$= \left(\frac{mz_2}{2}\right)^2 \left[1 + \left(\frac{1}{i^2}\right)\sin^2\alpha + \left(\frac{2}{i}\right)\sin^2\alpha\right]$$

$$O_2A = \left(\frac{mz_2}{2}\right)\left[1 + \left(\frac{1}{i^2}\right)\sin^2\alpha + \left(\frac{2}{i}\right)\sin^2\alpha\right]^{0.5} \quad (11.22)$$

Also

$$O_2A = O_2P + PC$$

$$= \frac{mz_2}{2} + h_{a2}$$

$$= \frac{mz_2}{2} + a_w m \tag{11.23}$$

From Eqs. (11.22) and (11.23), we get

$$\frac{mz_2}{2} + a_w m = \left(\frac{mz_2}{2}\right)\left[1 + \left(\frac{1}{i^2}\right)\sin^2\alpha + \left(\frac{2}{i}\right)\sin^2\alpha\right]^{0.5}$$

or

$$a_w = \left(\frac{z_2}{2}\right)\left[\left\{1 + \left(\frac{1}{i^2}\right)\sin^2\alpha + \left(\frac{2}{i}\right)\sin^2\alpha\right\}^{0.5} - 1\right]$$

or

$$z_2 = \frac{2a_w}{\left[1 + \left(\frac{1}{i^2}\right)\sin^2\alpha + \left(\frac{2}{i}\right)\sin^2\alpha\right]^{0.5} - 1} \tag{11.24}$$

or

$$z_2 = \frac{z_1^2\sin^2\alpha - 4a_w^2}{4a_w - 2z_1\sin^2\alpha} \tag{11.25}$$

For

$$a_w = 1,\ z_2 = \frac{z_1^2\sin^2\alpha - 4}{4 - 2z_1\sin^2\alpha} \tag{11.26}$$

For

$$i = 1,\ a_w = a_p$$

and

$$z_2 = \frac{2a_w}{\left[1 + 3\sin^2\alpha\right]^{0.5} - 1} \tag{11.27}$$

11.14.2 Pinion

From ΔBO_1P, we have

$$O_1B_2 = O_1P^2 + BP^2 - 2\cdot O_1P\cdot BP\cdot\cos\angle O_1PB$$

$$= \left(\frac{mz_1}{2}\right)^2 + (O_2P\sin\alpha)^2 - 2\cdot\frac{mz_1}{2}\cdot O_2P\sin\alpha\cdot\cos(90^\circ + \alpha)$$

$$= \left(\frac{mz_1}{2}\right)^2 + \left(\frac{mz_2}{2}\right)^2\sin^2\alpha + 2\cdot\left(\frac{mz_1}{2}\right)\cdot\left(\frac{mz_2}{2}\right)\sin^2\alpha$$

$$= \left(\frac{mz_1}{2}\right)^2\left[1 + i^2\sin^2\alpha + 2i\sin^2\alpha\right]$$

$$O_1B = \left(\frac{mz_1}{2}\right)\left[1 + i^2\sin^2\alpha + 2i\sin^2\alpha\right]^{0.5} \tag{11.28}$$

Also

$$O_1B = O_1P + PD$$

$$= \frac{mz_1}{2} + h_{a1}$$

$$= \frac{mz_1}{2} + a_p m \tag{11.29}$$

From Eqns. (11.28) and (11.29), we get

$$\frac{mz_1}{2} + a_p m = \left(\frac{mz_1}{2}\right)\left[1 + i^2\sin^2\alpha + 2i\sin^2\alpha\right]^{0.5} \tag{11.30}$$

or

$$a_p = \left(\frac{z_1}{2}\right)\left[\left\{1 + i^2\sin^2\alpha + 2i\sin^2\alpha\right\}^{0.5} - 1\right]$$

or

$$z_1 = \frac{2a_p}{\left[1 + i^2\sin^2\alpha + 2i\sin^2\alpha\right]^{0.5} - 1} \tag{11.31}$$

or $$z_1 = \frac{z_2^2 \sin^2\alpha - 4a_p^2}{4a_p - 2z_2 \sin^2\alpha} \quad (11.32)$$

For $$a_p = 1,\ z_1 = \frac{z_2^2 \sin^2\alpha - 4}{4 - 2z_2 \sin^2\alpha} \quad (11.33)$$

For $\alpha = 14.5°$,

$$(z_1)_{\min} = \frac{z_2^2 - 63.8}{63.8 - 2z_2} \quad (11.34)$$

For $\alpha = 20°$,

$$(z_1)_{\min} = \frac{z_2^2 - 34.2}{34.2 - 2z_2} \quad (11.35)$$

11.14.3 Rack and Pinion

A rack is a gear of infinite pitch radius. Thus its pitch circle is a straight line, called the *pitch line*. The line of action is tangential to the base circle at infinity; hence the involute profile of the rack is a straight line and is perpendicular to the line of action. For the rack and pinion shown in Fig.11.17, let

Addendum of rack, $h_r = a_r m$

$$h_r = AB = AP\sin\alpha$$
$$= O_1P\sin\alpha\sin\alpha$$
$$= O_1P\sin^2\alpha$$
$$= r_1\sin^2\alpha$$

or $$a_r m = \left(\frac{mz_1}{2}\right)\sin^2\alpha$$

or $$a_r = \left(\frac{z_1}{2}\right)\sin^2\alpha$$

or $$z_1 = \frac{2a_r}{\sin^2\alpha} \quad (11.36)$$

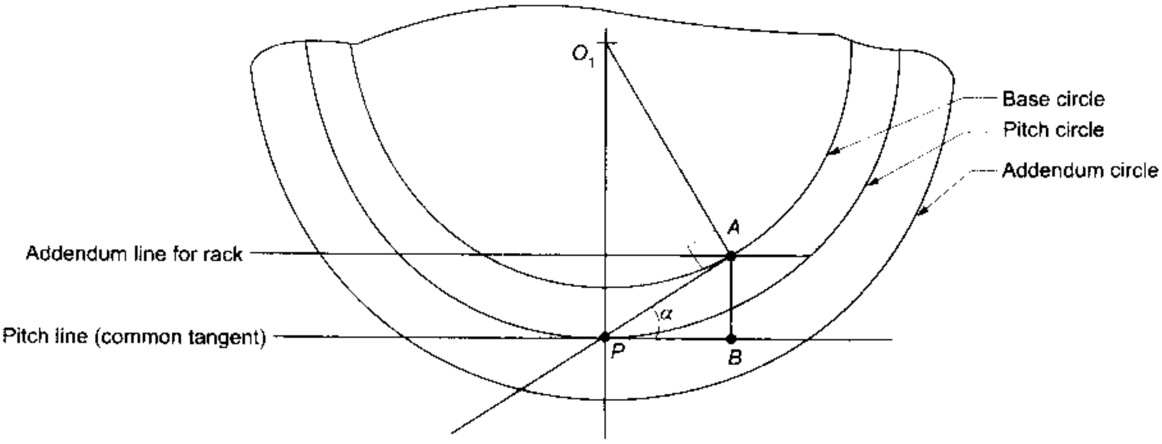


Fig.11.17 Minimum number of teeth on rack and pinion

For $a_r = 1$, the minimum number of teeth on the pinion are given in Table 11.2.

Table 11.2 Minimum number of teeth on the pinion for a rack.

α	14.5°	20°	20° stub	25°
$(z_1)_{min}$	32	18	14	12

Example 11.8

Determine the minimum number of teeth on the 20° pinion in order to avoid interference with a gear to give a gear ratio of 3:1. The addendum on wheel is equal to one module.

■ **Solution**

$$h_{a2} = a_w m,\ 1m = a_w m,\ \text{or } a_w = 1$$

$$z_2 = \frac{2a_w}{\left[1 + \left(\frac{1}{i^2}\right)\sin^2\alpha + \left(\frac{2}{i}\right)\sin^2\alpha\right]^{0.5} - 1}$$

$$z_1 = \frac{2a_w}{i\left[1 + \left(\frac{1}{i^2}\right)\sin^2\alpha + \left(\frac{2}{i}\right)\sin^2\alpha\right]^{0.5} - 1}$$

$$= \frac{2 \times \frac{1}{3}}{\left[1 + \left(\frac{1}{9}\right)\sin^2 20° + \left(\frac{2}{3}\right)\sin^2 20°\right]^{0.5} - 1}$$

$$= 14.98 \approx 15$$

Example 11.9

A pinion of 20° involute teeth and 120 mm pitch circle diameter drives a rack. The addendum of both pinion and rack is 6 mm. Determine the least pressure angle which can be used to avoid interference. With this pressure angle find the contact ratio.

■ **Solution**

$$h_a = r_1 \sin^2\alpha$$

$$6 = 60\sin^2\alpha$$

$$\alpha = 18.435°$$

$$r_{a1} = r_1 + h_a = 60 + 6 = 66 \text{ mm}$$

$$r_{b1} = r_1 \cos\alpha = 60\cos 18.435° = 56.92 \text{ mm}$$

Length of path of contact,

$$L_p = \left[r_{a1}^2 - r_{b1}^2\right]^{0.5} = \left[66^2 - 56.92^2\right]^{0.5}$$

$$= 33.4 \text{ mm}$$

Base pitch,

$$p_b = p\cos\alpha = \left(\frac{\pi d_1}{z_1}\right)\cos\alpha$$

$$= \left(\pi \times \frac{120}{20}\right)\cos 18.435° = 17.88 \text{ mm}$$

Minimum number of teeth in contact $= \dfrac{L_p}{p_b} = \dfrac{33.4}{17.88} = 1.87 \approx 2$

Example 11.10

Two 3 mm module, 20° pressure angle involute spur gears mesh externally to give a velocity ratio of 4. The addendum is 1.2 times the module. The pinion rotates at 150 rpm. Determine (a) the minimum number of teeth on each gear wheel to avoid interference and (b) the number of pairs of teeth in contact.

Solution

$$m = 3\text{ mm}, a_w = 1.2, h_a = 1.2 \times 3 = 3.6\text{ mm}, \alpha = 20°, i = 4, n_1 = 150\text{ rpm}$$

(a)

$$z_2 = \frac{2a_w}{\left[1 + \left(\frac{1}{i^2}\right)\sin^2\alpha + \left(\frac{2}{i}\right)\sin^2\alpha\right]^{0.5} - 1}$$

$$= \frac{2 \times 1.2}{\left[1 + \left(\frac{1}{16}\right)\sin^2 20° + \left(\frac{2}{4}\right)\sin^2 20°\right]^{0.5} - 1}$$

$$= \frac{2.4}{\left[1 + 7.311 \times 10^{-3} + 0.05849\right]^{0.5} - 1} = \frac{2.4}{1.03237 - 1}$$

$$= 74.13 \cong 76 \quad \text{so that } z_2 \text{ is divisible by 4.}$$

$$z_1 = \frac{76}{4} = 19$$

(b)

$$r_1 = \frac{mz_1}{2} = \frac{3 \times 19}{2} = 28.5\text{ mm}, r_2 = \frac{mz_2}{2} = \frac{3 \times 76}{2} = 114\text{ mm}$$

$$r_{a1} = r_1 + h_{a1} = 28.5 + 3.6 = 32.1\text{ mm}$$

$$r_{a2} = r_2 + h_{a2} = 114 + 3.6 = 117.6\text{ mm}$$

$$r_{b1} = r_1\cos\alpha = 28.5\cos 20° = 26.78\text{ mm}$$

$$r_{b2} = r_2\cos\alpha = 114\cos 20° = 107.12\text{ mm}$$

$$L_p = \left[r_{a1}^2 - r_{b1}^2\right]^{0.5} + \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - (r_1 + r_2)\sin\alpha$$

$$= \left[(32.1)^2 - (26.78)^2\right]^{0.5} + \left[(117.6)^2 - (107.12)^2\right]^{0.5}$$

$$-(28.5 + 114)\sin 20° = 17.49\text{ mm}$$

$$\text{Number of pairs of teeth in contact} = \frac{L_p}{\pi m\cos\alpha} = \frac{17.49}{\pi \times 3 \times \cos 20°}$$

$$= 1.975 \cong 2$$

11.15 GEAR STANDARDIZATION

A set of gears is interchangeable when any two gears selected from the set will mesh and satisfy the fundamental law of gearing. For interchangeability all gears of the set must have the same circular pitch, module, diametral pitch, pressure angle, addendum and dedendum; and the tooth thickness must be half of the circular pitch. Standard tooth forms ensure readily availability of gears.

The standard pressure angles are 14.5° and 20°. The 20° full-depth system has several advantages when compared with the 25° or 30° full-depth system. The lower pressure angle gives a higher contact ratio which results in quieter operation, reduced wear, reduced tooth load and reduced bearing loads.

The larger pressure angle tooth forms results in broader teeth at the base and hence are stronger in bending. Also fewer teeth may be used on the pinion without undercutting the teeth. The proportions of standard tooth forms are given in Table 11.3.

Table 11.3 Standard tooth forms

	14.5° full-depth	20° full-depth	20° stub
Addendum, h_a	m	m	0.800 m
Dedendum, h_f	1.157 m	1.250 m	m
Clearance, c	0.157 m	0.250 m	0.200 m
Fillet radius, r	0.209 m	0.300 m	0.304 m
Tooth thickness, t	1.5708 m	1.5708 m	1.5708 m

11.16 EFFECT OF CENTRE DISTANCE VARIATION ON VELOCITY RATIO

Consider a pair of teeth in contact at L, as shown in Fig.11.18. The angular velocity ratio is,

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$

Let the centre distance of rotation of gear 2 be shifted from O_2 to O'_2. As a result of this change, the contact point will shift to L'. Common normal at the point of contact L' is tangent to the base circle, because it is in contact between two involute curves, and they are generated from the base circle. Let the tangent to the base circle $M'N'$ intersect the line joining the centers of rotation O' and O_1 at P'.

Triangles O_1PN and O_2MP are similar. Also triangles $O_1N'P'$ and $O'_2M'P'$ are similar. Therefore,

$$\frac{MO'_2}{N'O_1} = \frac{O'_2P'}{O_1P'}$$

and $$\frac{MO_2}{NO_1} = \frac{O_2P}{O_1P}$$

But $$NO_1 = N'O_1 \quad \text{and} \quad O_2M = O'_2M'$$

Therefore, $$\frac{O_2P}{O_1P} = \frac{O'_2P'}{O_1P'}$$

Hence, the variation in the centre distance, within limits, does not affect the angular velocity ratio. But the length of arc of contact is decreased, and the pressure angle is increased.

11.17 DETERMINATION OF BACKLASH

Two standard gears in mesh are shown in Fig.11.19(a). The standard centre distance with zero backlash is,

$$C = \frac{m\,(z_1 + z_2)}{2}$$

The cutting pitch circles are known as standard pitch circles. Fig.11.19(b) shows the condition where the two gears have been pulled apart a distance Δc to give a new centre distance C'. The line of action now crosses the line of centers at a new pitch point P'. The standard pitch radii r_1 and r_2 are now no longer tangent to each other. The pitch point P' divides the centre distance C' into segments which are inversely proportional to the angular velocity ratio. These segments become the radii r'_1 and r'_2 of new pitch circles that are tangent to each other at point P'. These circles are known as operating pitch circles.

$$\frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{r'_2}{r'_1}$$

and $$C' = r'_1 + r'_2$$

to give $$r'_1 = \left[\frac{z_1}{z_1 + z_2}\right] C'$$

and $$r'_2 = \left[\frac{z_2}{z_1 + z_2}\right] C'$$

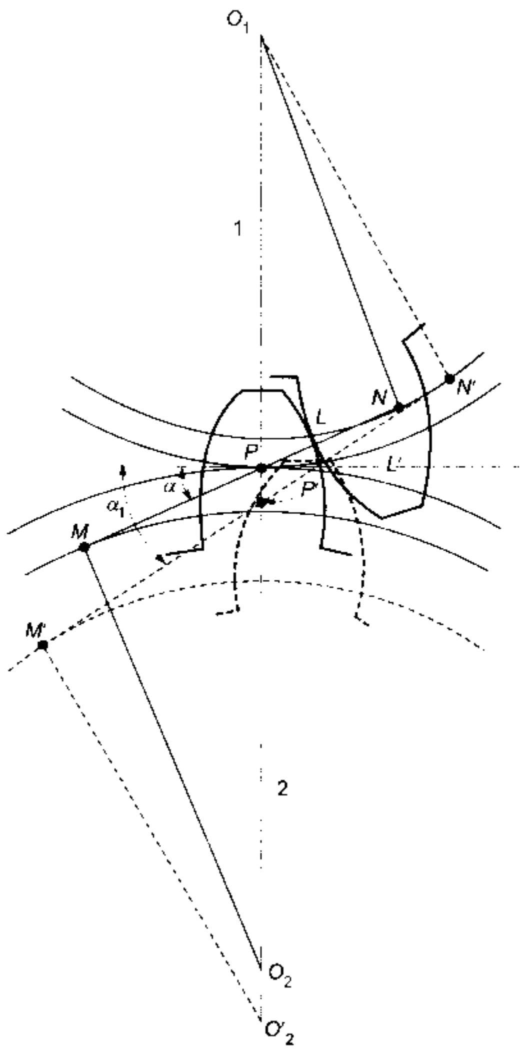


Fig.11.18 Effect of centre distance variation

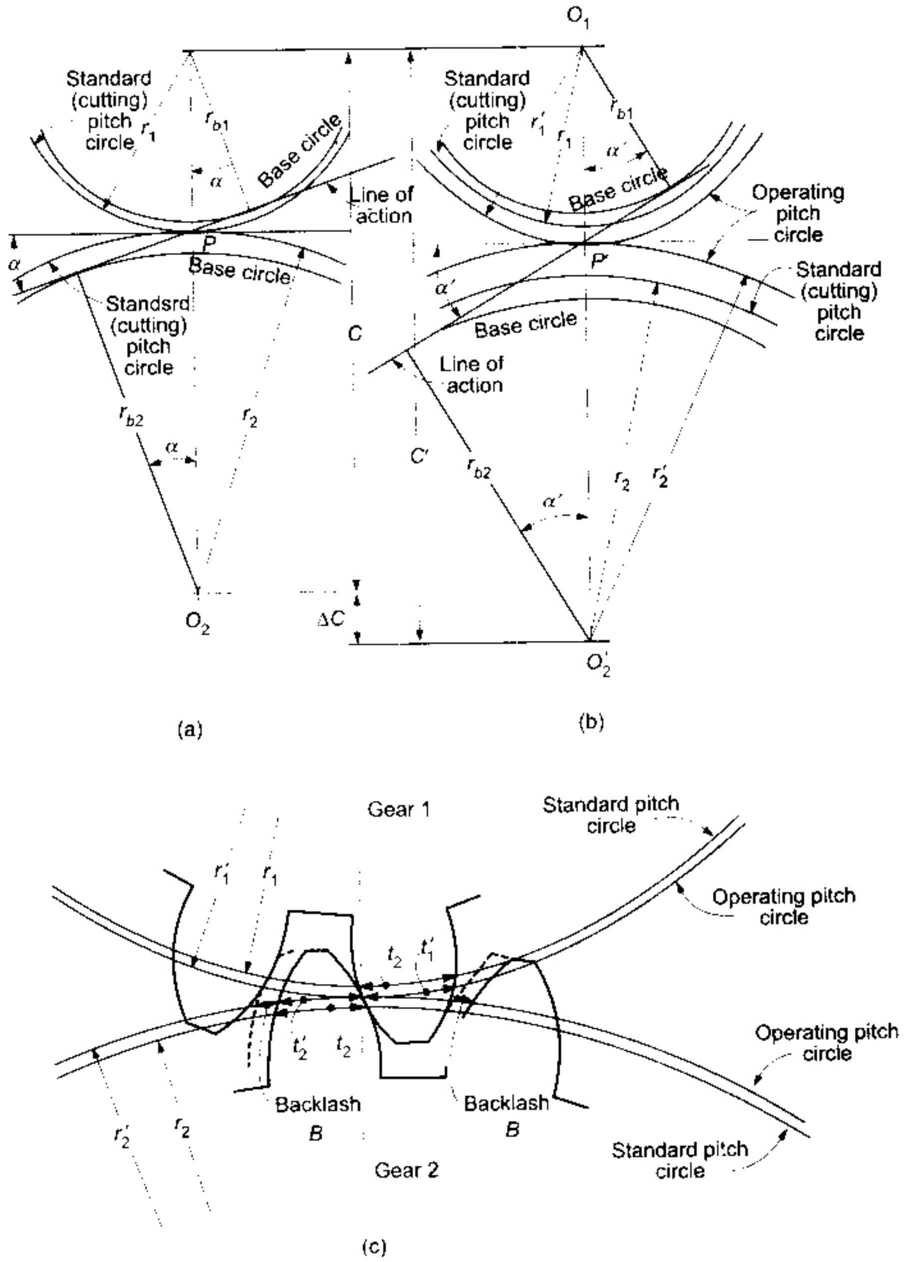


Fig.11.19 Determination of backlash

Let α' be the operating pressure angle. Now

$$C' = \frac{r_{b1} + r_{b2}}{\cos\alpha'} = \frac{(r_1 + r_2)\cos\alpha}{\cos\alpha'}$$

$$= \frac{C\cos\alpha}{\cos\alpha'}$$

or $$\cos\alpha' = \frac{C\cos\alpha}{C'} \tag{11.37}$$

Also $$\Delta C = C' - C$$
$$= \frac{C\cos\alpha}{\cos\alpha'} - C \tag{11.38}$$
$$= C\left[\frac{\cos\alpha}{\cos\alpha'} - 1\right] \tag{11.39}$$

Now from Fig.11.19(c),

$$t_1' + t_2' + B = \frac{2\pi r_1'}{z_1} = \frac{2\pi r_2'}{z_2} \tag{11.40}$$

where t' = tooth thickness on operating pitch circle
B = backlash
r' = radius of operating pitch circle
z = number of teeth

Now $$t_1' = 2r_1'\left[\frac{t_1}{2r_1} + \text{inv}(\alpha) - \text{inv}(\alpha')\right]$$
$$= \frac{r_1' t_1}{r_1} - 2r_1'\left[\text{inv}(\alpha) - \text{inv}(\alpha')\right] \tag{11.41}$$
$$t_2' = 2r_2'\left[\frac{t_2}{2r_2} + \text{inv}(\alpha) - \text{inv}(\alpha')\right] \tag{11.42}$$
$$= \frac{r_2' t_2}{r_2} - 2r_2'\left[\text{inv}(\alpha) - \text{inv}(\alpha')\right] \tag{11.43}$$

where t = tooth thickness on standard pitch circle
$= \frac{p}{2} = \frac{\pi m}{2}$
r = radius of standard pitch circle
$= \frac{mz}{2}$

Also $$\frac{r_1}{r_1'} = \frac{r_2}{r_2'} = \frac{C}{C'} \tag{11.44}$$

and $$C' = r_1' + r_2' \tag{11.45}$$

Substituting (11.41) to (11.45) in Eq. (11.39), we get

$$B = \left(\frac{C'}{C}\right)\left[\pi m - (t_1 + t_2) + 2C\left\{\text{inv}(\alpha) - \text{inv}(\alpha')\right\}\right] \tag{11.46}$$
$$= 2C\left[\text{inv}(\alpha) - \text{inv}(\alpha')\right] \tag{11.47}$$

Example 11.11

A three-module, 20° pinion of 24 teeth drives a gear of 60 teeth. (a) Calculate the length of action and contact ratio, if the gears mesh with zero backlash. (b) If the centre distance is increased 0.5 mm, calculate the radii of the operating pitch circles, the operating pressure angle and the backlash produced.

■ Solution

(a)

$$r_1 = \frac{z_1 m}{2} = 24 \times \frac{3}{2} = 36 \text{ mm}$$

$$r_2 = \frac{z_2 m}{2} = 60 \times \frac{3}{2} = 90 \text{ mm}$$

$$r_{b1} = r_1 \cos\alpha = 36 \times \cos 20^\circ = 33.83 \text{ mm}$$

$$r_{b2} = r_2 \cos\alpha = 90 \times \cos 20^\circ = 84.57 \text{ mm}$$

$$a_1 = a_2 = m = 3 \text{ mm}$$

$$r_{a1} = r_1 + a_1 = 36 = 3 = 39 \text{ mm}$$

$$r_{a2} = r_2 + a_2 = 90 + 3 = 93 \text{ mm}$$

$$C = r_1 + r_2 = 36 + 90 = 126 \text{ mm}$$

Length of path of contact,

$$AB = \left[r_{a1}^2 - r_{b1}^2\right]^{0.5} + \left[r_{a2}^2 - r_{b2}^2\right]^{0.5} - (r_1 + r_2)\sin\alpha$$

$$= \left(39^2 - 33.83^2\right)^{0.5} + \left(93^2 - 84.57^2\right)^{0.5} - 126 \sin 20^\circ$$

$$= 14.997 \text{ mm}$$

Contact ratio,

$$m_c = \frac{AB}{p_b}$$

$$p_b = 2\pi \times \frac{33.83}{24} = 8.856 \text{ mm}$$

$$m_c = \frac{14.997}{8.856} = 1.693$$

(b)

$$C' = C + \Delta C = 126 + 0.5 = 126.5 \text{ mm}$$

$$r_1' = \left[\frac{z_1}{z_1 + z_2}\right] C' = \left[\frac{24}{84}\right] 126.5 = 36.143 \text{ mm}$$

$$r_2' = C' - r_1' = 126.5 - 36.143 = 90.357 \text{ mm}$$

$$\cos\alpha' = \frac{C\cos\alpha'}{C'} = \frac{126\cos 20^\circ}{126.5} = 0.93598$$

$$\alpha' = 20.61^\circ$$

Backlash,

$$B = 2C'\left[\text{inv}(\alpha') - \text{inv}(\alpha)\right]$$

$$= 2 \times 126.5[\text{inv}(20.61^\circ) - \text{inv}(20^\circ)]$$

$$= 253[0.016362 - 0.014904]$$

$$= 0.3689 \text{ mm}$$

11.18 INTERNAL GEARS

A pinion in mesh with an internal gear is shown in Fig.11.20. Internal gears have some advantages over the external gears. The most important advantage is the compactness of the drive. Other advantages are greater length of contact, greater tooth strength and lower relative sliding velocity between meshing teeth.

The tooth profile is concave in internal gears instead of convex as in external gears. Because of this a type of interference called *fouling* may occur in internal gears. Fouling occurs between inactive profiles as the teeth go in and out of the mesh and there is not sufficient difference between the numbers of teeth on the internal gear and the pinion.

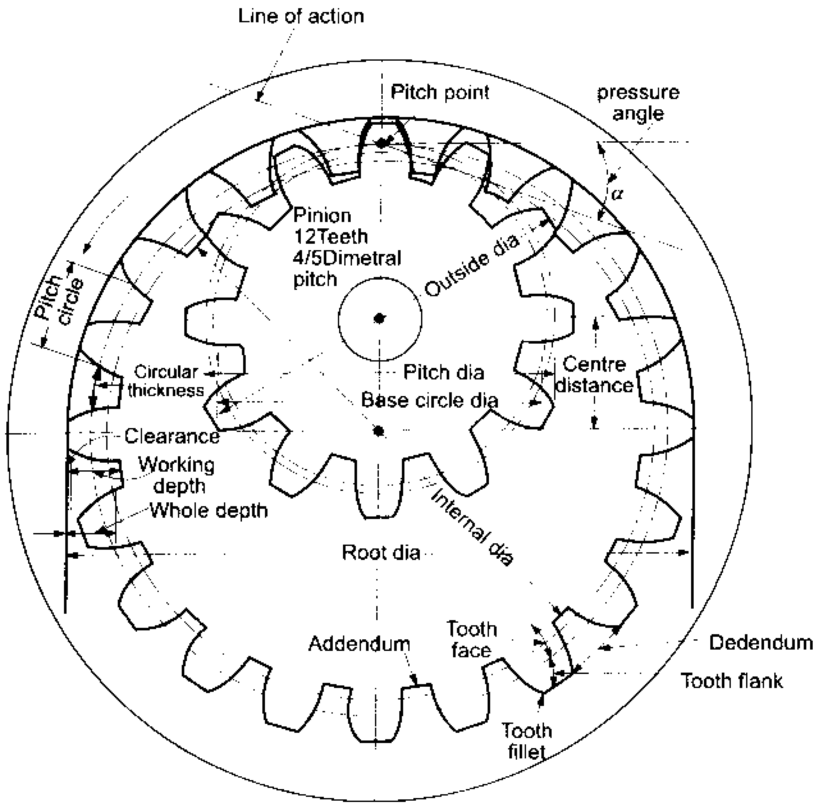


Fig.11.20 Internal gears

11.19 HELICAL GEARS

If a plane is rolled on a base cylinder, a line in the plane parallel to the axis of the cylinder will generate the surface of an involute spur gear tooth. If the generating line is inclined to the axis, the surface of a helical gear tooth will be generated. These two conditions are shown in Fig.11.21.

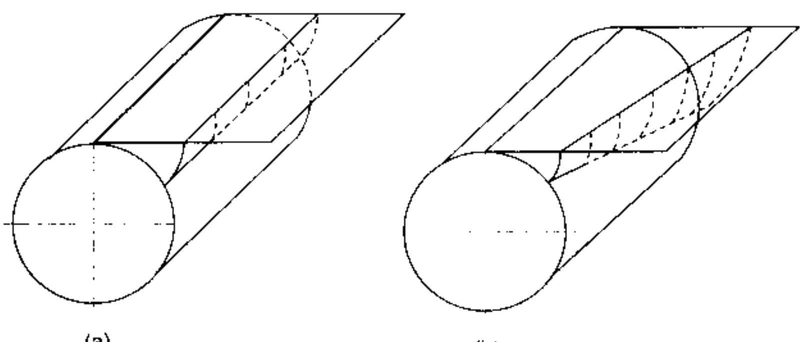


Fig.11.21 Generation of helical gears

Helical gears are used to connect parallel shafts and non-intersecting shafts. The former are known as *parallel helical gears* and the latter as *crossed helical gears*, as shown in Fig.11.22.

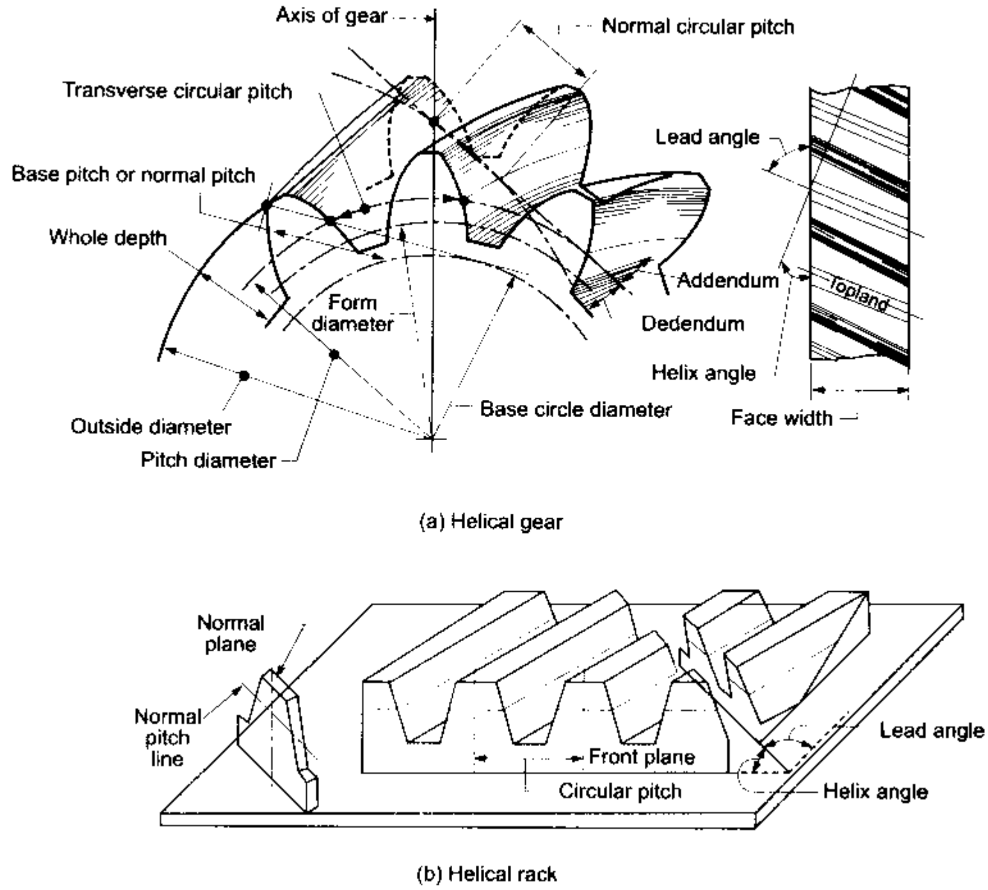


Fig.11.22 Helical gear and rack terminology

In determining the tooth proportions of a helical gear for either crossed or parallel shafts, it is necessary to consider the manner in which the teeth are to be cut. If the gear is to be hobbed, all dimensions are figured in a plane that is normal to the tooth pitch element, and the diametral pitch and the pressure angle are standard values in that plane. As the cutting action of the hob occurs in the normal plane, it is possible to use the same hob to cut both helical and spur gears of a given pitch; in a spur gear the normal plane and the plane of rotation (or transverse plane) are identical.

Helical gears connecting parallel shafts have line contact which runs diagonally across the face of the tooth. Parallel helical gears have smoother action and hence less noise and vibration than spur gears. Also the tooth contact is gradual, beginning at one end of the tooth and progressing across the tooth surface, whereas in spur gears, contact takes place simultaneously over the entire face width. However, helical gears give rise to end thrust. The various pitches are shown in Fig.11.23.

Helix angle (β) The helix angle is the angle between a line drawn through one of the teeth and the centre line of the shaft on which the gear is mounted.

Normal circular pitch (p_n) The normal circular pitch is the distance between corresponding points of adjacent teeth as measured in a plane perpendicular (or normal) to the helix. It is the perpendicular distance between two adjacent teeth.

$$p_n = p_t \cos\beta = \pi m \cos\beta \qquad (11.48)$$

Normal diametral pitch (P_n) The normal diametral pitch is the diametral pitch measured in the plane normal to the helix. It is equal to the diametral pitch of the *hob*.

$$P_n = \frac{P_t}{\cos\beta} \qquad (11.49)$$

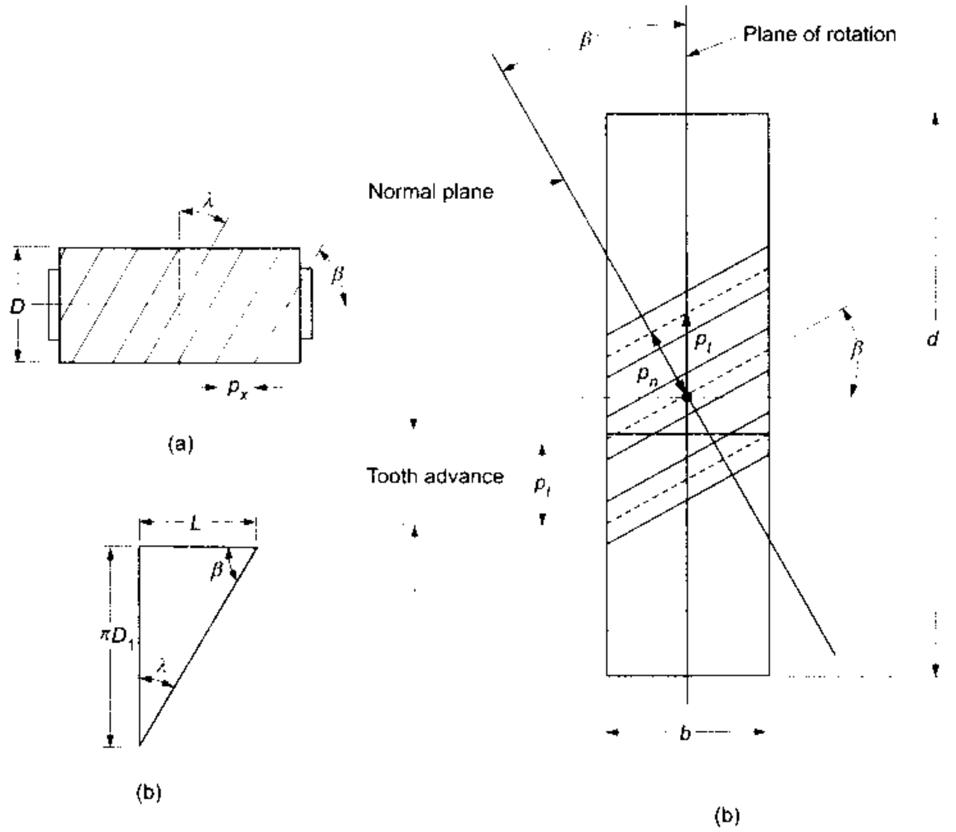


Fig.11.23 Definition of various pitches

Transverse circular pitch (p_t) The transverse circular pitch is the distance measured in a plane perpendicular to the shaft axis (or plane of rotation) between corresponding points of adjacent teeth.

$$p_t = \frac{\pi d}{z} = \pi m \qquad (11.50)$$

Transverse diametral pitch (P_t) The transverse diametral pitch is the diametral pitch measured in the plane of rotation, that is, transverse to the axis of rotation.

$$P_t = \frac{z}{d} = \frac{1}{m} \qquad (11.51)$$

$$P_t p_t = P_n p_n = \pi \qquad (11.52)$$

Axial pitch (p_x) The axial pitch is the distance measured in a plane parallel to the shaft axis between corresponding points of adjacent teeth.

$$p_x = p_t \cot \beta \tag{11.53}$$

Lead The lead is the distance measured parallel to the axis to represent the distance advanced by each tooth per revolution.

Lead angle The lead angle is the acute angle between the tangent to the helix and a plane perpendicular to the axis of cylinder.

Virtual (or formative or equivalent) number of teeth (z_v) The number of teeth of the equivalent spur gear in the normal plane are called virtual number of teeth.

$$z_v = \frac{z}{\cos^3 \beta} \tag{11.54}$$

Normal module (m_n) The normal module is the module measured in a plane normal to the helix.

Normal module, $$m_n = m_t \cos \beta \tag{11.55}$$

Pitch diameter

$$d = \frac{z m_n}{\cos \beta} \tag{11.56}$$

Forces in helical gears The forces in a helical gear are shown in Fig.11.24. Pressure angle in the plane of rotation,

Tangential force, $$F_t = F_n \cos \alpha_n \cos \beta \tag{11.57}$$

$$= \frac{10^3 P}{v_m} \text{ N}$$

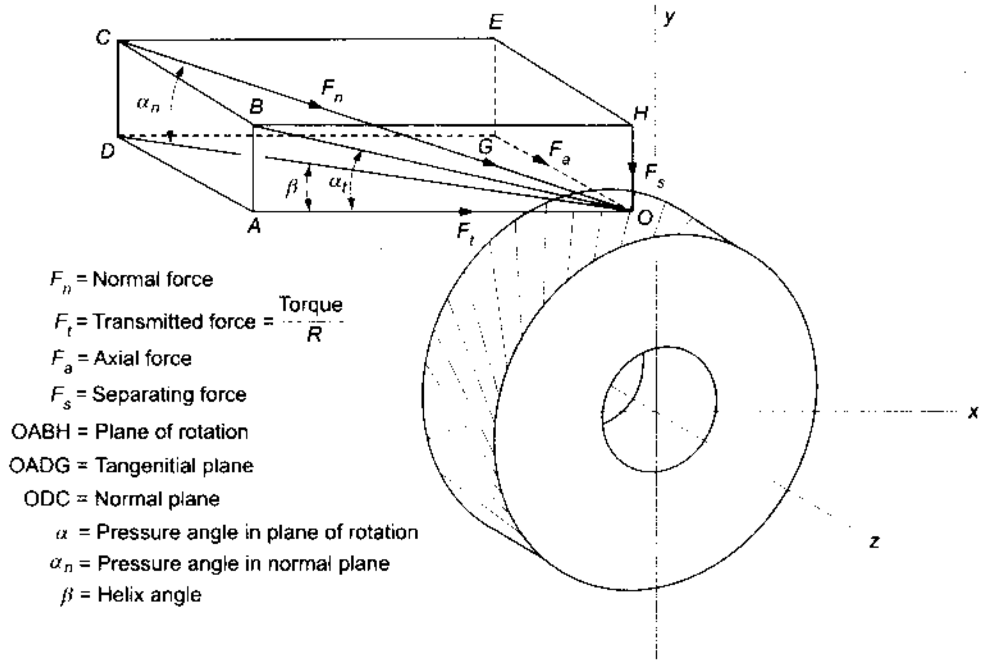


Fig.11.24 Forces on a helical gear

where F_n = normal force on gear tooth

P = power transmitted in kW

v_m = mean speed of gear pair

$$\tan \alpha_t = \frac{F_r}{F_t} \text{ (plane } OABH)$$

where F_r = radial force

$$\tan \alpha_n = \frac{F_r}{OD} \text{ (plane } ODC)$$

$$OD = \frac{F_t}{\cos \beta} \text{ (plane } OADG)$$

Therefore,
$$\tan \alpha_n = \left(\frac{F_r}{F_t}\right) \cos \beta$$

$$\tan \alpha_n = \tan \alpha_t \cos \beta \quad (11.58)$$

Axial force,
$$F_a = F_t \tan \beta \quad (11.59)$$

The minimum number of teeth cut by a *hob*,

$$z_{\min} = \frac{2k \cos \beta}{\sin^2 \alpha_t} \quad (11.60)$$

where addendum = $k \cdot m$, and k is a constant. For full depth teeth, k = 1.00; and for the stub system, $k = 0.80$.

If the gear is to be cut by a gear shaper method, the dimensions are considered in the plane of rotation and the diametral pitch and the pressure angle are standard values in that plane. When a helical gear is cut by a gear shaper, the circular pitch p_t of Fig.11.23 becomes equal to the circular pitch of the cutter so that the following relations apply:

$$p_t = \frac{\pi d}{z} = \frac{\pi}{P_t} = \pi m \quad (11.61)$$

$$P_t = \frac{z}{d} \quad (11.62)$$

$$m_t = \frac{d}{z} \quad (11.63)$$

11.19.1 Parallel Helical Gears

For parallel helical gears to mesh properly, the following conditions must be satisfied:

1. Equal helix angles
2. Equal pitches or modules
3. Opposite hand of helices

Velocity ratio,
$$\frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{d_2 \cos \beta_2}{d_1 \cos \beta_1}$$

$$= \frac{d_2}{d_1} \quad (11.64)$$

Centre distance,
$$C = (z_1 + z_2) \frac{m_t}{2} \quad (11.65)$$

11.19.2 Crossed Helical Gears

For crossed helical gears to mesh properly, there is only one requirement that they must have common normal pitches or modules. Their pitches in the plane of rotation are not necessarily equal. Their helix angles may or may not be equal and the gears may be of the same or of opposite hand.

Velocity ratio,

$$\frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{d_2 \cos \beta_2}{d_1 \cos \beta_1} \tag{11.66}$$

The angle between the two shafts,

$$\Sigma = \beta_1 \pm \beta_2 \tag{11.67}$$

The plus and minus signs apply respectively, when the gears have the same or the opposite hand. Fig. 11.25 illustrates pairs of crossed helical gears in and out of mesh.

Centre distance,

$$C = \left(\frac{m_n}{2}\right)\left[\frac{z_1}{\cos \beta_1} + \frac{z_2}{\cos \beta_2}\right] \tag{11.68}$$

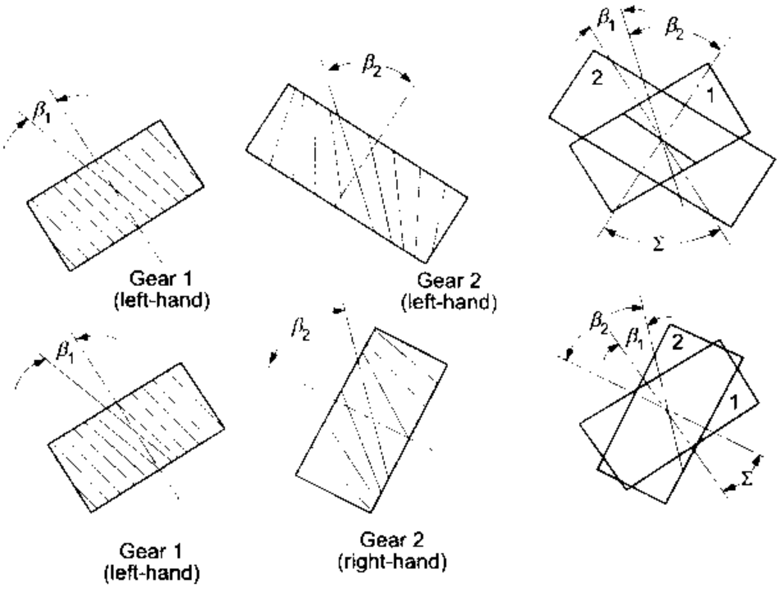


Fig.11.25 Helical gear in and out of mesh

Example 11.12

A pair of crossed helical gears connect two shafts at an angle of 60° with a velocity ratio of 1.5:1. The pinion has a normal diametral pitch of 0.25, a pitch diameter of 200 mm and a helix angle of 35°. Determine the helix angle and the pitch diameter of the gear and the number of teeth on both the pinion and the gear.

■ Solution

$$\Sigma = \beta_1 + \beta_2$$

Here $\Sigma = 60°$ and $\beta_1 = 35°$. Therefore, $\beta_2 = 25°$.

Now
$$\frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{d_2 \cos\beta_2}{d_1 \cos\beta_1}$$
$$d_2 = 200 \times \frac{1.5 \cos 35^\circ}{\cos 25^\circ} = 271.15 \text{ mm}$$
$$z_1 = p_n d_1 \cos\beta_1 = 0.25 \times 200 \times \cos 35^\circ = 40.95 \approx 41$$
$$z_2 = 41 \times 1.5 = 61.6 \approx 62$$

Example 11.13

Two left handed helical gears connect two shafts inclined at 60°. The normal module is 8 mm. The larger gear has 72 teeth and the velocity ratio is 1:2. If centre distance is 500 mm, calculate the helix angles of the two gears.

■ **Solution**

Here
$$\Sigma = 60^\circ, m_n = 8 \text{ mm}, z_2 = 72, C = 500 \text{ mm}$$
$$\Sigma = \beta_1 + \beta_2$$
$$\beta_2 = 60^\circ - \beta_1$$
$$i = \frac{n_2}{n_1} = \frac{z_1}{z_2}$$
$$z_1 = \frac{72}{2} = 36$$
$$C = \left(\frac{m_n}{2}\right)\left[\frac{z_1}{\cos\beta_1} + \frac{z_2}{\cos\beta_2}\right]$$
$$500 = \left(\frac{8}{2}\right)\left[\frac{36}{\cos\beta_1} + \frac{72}{\cos(60^\circ - \beta_1)}\right]$$
or
$$3.472 = \frac{1}{\cos\beta_1} + \frac{2}{\cos(60^\circ - \beta_1)}$$

Table 11.4

β_1, deg	$\frac{1}{\cos\beta_1}$	$\frac{2}{\cos(60^\circ - \beta_1)}$	$\frac{1}{\cos\beta_1} + \frac{2}{\cos(60^\circ - \beta_1)}$
25	1.1034	2.4415	3.5449
27	1.1223	2.3847	3.5070
28	1.1325	2.3583	3.4908
29	1.1433	2.3332	3.4765
30	1.1547	2.3094	3.4641

We take $\beta_1 = 29^\circ$, then $\beta_2 = 31^\circ$.

11.19.3 Herringbone Gears

It is a gear, half of whose width is cut with a tooth helix in one direction and the other half in the opposite direction. It is in effect a double helical gear cut on a blank. The advantage of herringbone gears is that the end thrust is automatically eliminated.

11.20 BEVEL GEARS

Bevel gears are used to connect shafts whose axes intersect. The shaft angle is defined as the angle between the centre lines which contains the engaging teeth. Figure 11.26 shows the details of a pair of bevel gears.

Pitch cone The pitch cone is the pitch surface of a bevel gear in a gear pair.

Cone centre The cone centre is the apex of the pitch cone.

Pitch cone radius (r) The pitch cone radius is the length of the pitch cone element.

Pitch angle (δ) The pitch angle is the angle that the pitch line makes with the axis of the gear.

Reference cone angle The reference cone angle is the angle between the gear axis and the reference cone generator containing the root cone generator.

Tip (or face) angle (δ_a) The tip angle is the angle between the tip cone generator and the axis of the gear.

Root (or cutting) angle (δ_f) The root angle is angle between the root cone generator and the axis of the gear.

Back cone The back cone is an imaginary cone the elements of which are perpendicular to the elements of the pitch cone at the larger end of the tooth.

Gear diameter The gear diameter is the diameter of the largest pitch circle.

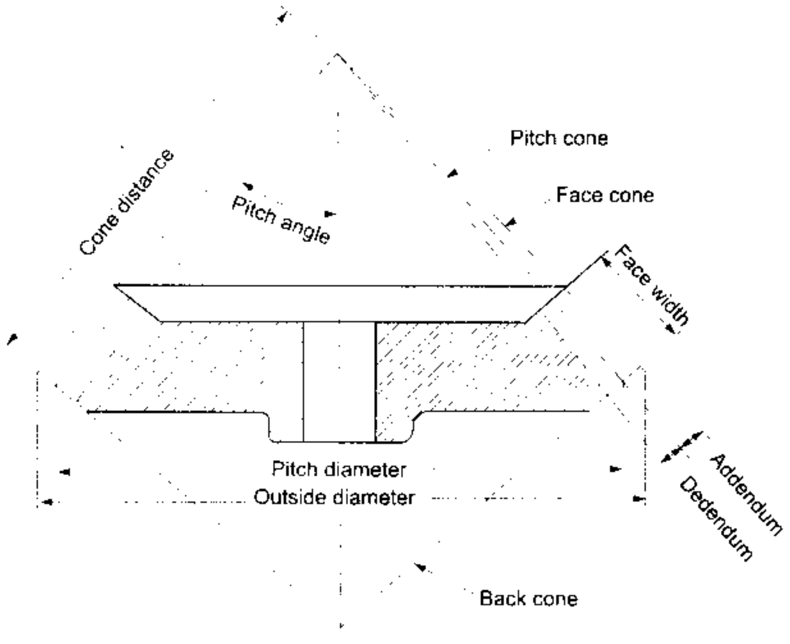
Virtual number of teeth The virtual number of teeth is number of teeth on an imaginary spur gear laid out on a pitch radius equal to the back cone radius.

$$z_v = \frac{z}{\cos \delta} \tag{11.69}$$

Crown gears The crown gear is a gear pair for which pitch cone angle is 90°.

Miter gears Mitre gears are two bevel gears of the same size having pitch cone angle 90°.

Angular bevel gears The shaft angle is greater or less than 90°.



(a) Bevel gear terminology

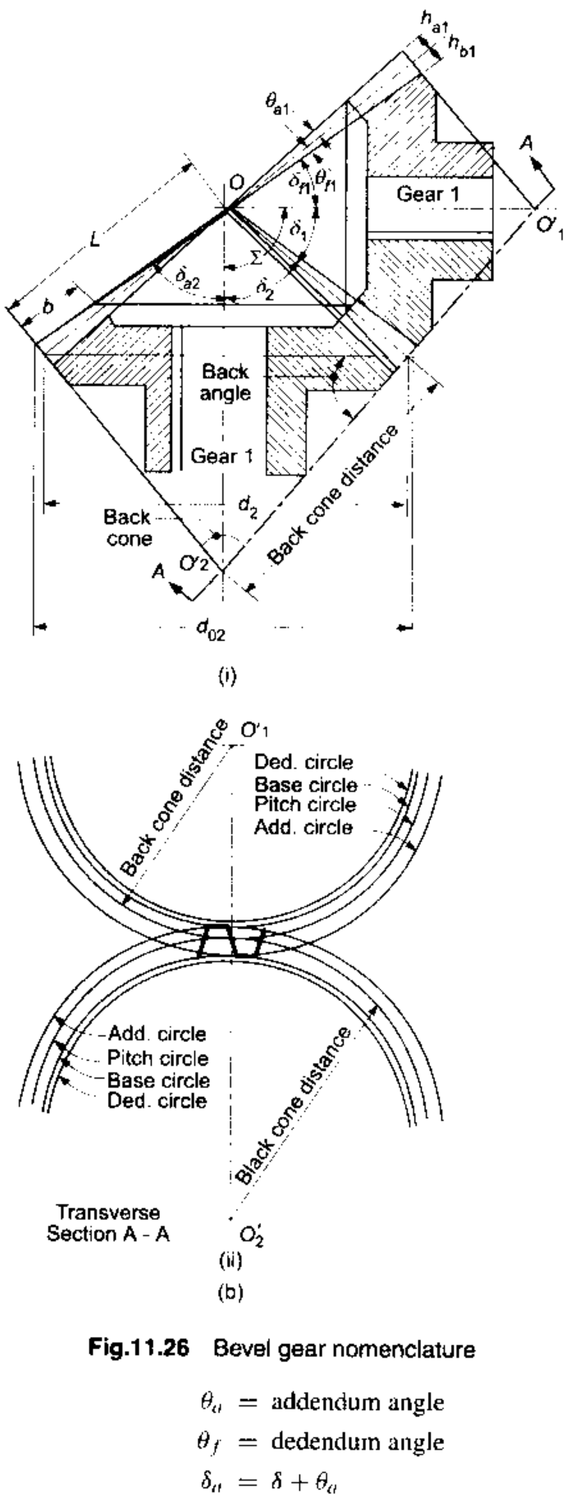


Fig.11.26 Bevel gear nomenclature

Let θ_a = addendum angle

θ_f = dedendum angle

Tip angle, $\delta_a = \delta + \theta_a$

$\delta_f = \delta - \theta_f$

Velocity ratio, $\frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1}$

Module, $m = \frac{d}{z}$

Angle relationships:

Let L = length of the pitch cone

$$= \left[\frac{d_1^2 + d_2^2}{4}\right]^{0.5} \tag{11.70}$$

$$\sin\delta_1 = \frac{d_1}{2L} = \sin(\Sigma - \delta_2) = \sin\Sigma\cos\delta_2 - \cos\Sigma\sin\delta_2$$

$$\frac{\sin\delta_1}{\sin\Sigma\sin\delta_2} = \frac{\cos\delta_2}{\sin\delta_2} - \frac{\cos\Sigma}{\sin\Sigma}$$

$$\left(\frac{1}{\sin\Sigma}\right)\left[\frac{\sin\delta_1}{\sin\delta_2} + \cos\Sigma\right] = \frac{1}{\tan\delta_2} \tag{11.71}$$

Also $\frac{\sin\delta_1}{\sin\delta_2} = \frac{d_1}{d_2}$

Therefore $\tan\delta_2 = \frac{\sin\Sigma}{\cos\Sigma + \frac{d_1}{d_2}} = \frac{\sin\Sigma}{\cos\Sigma + \frac{z_1}{z_2}}$

Similarly $\tan\delta_1 = \frac{\sin\Sigma}{\cos\Sigma + \frac{d_2}{d_1}}$

$$= \frac{\sin\Sigma}{\cos\Sigma + \frac{z_2}{z_1}} \tag{11.72}$$

Addendum angle, $\tan\theta_a = 2h_a \sin\frac{\delta_1}{d_1}$

$$= 2h_a \sin\frac{\delta_2}{d_2} \tag{11.73}$$

Dedendum angle, $\tan\theta_f = 2h_f \sin\frac{\delta_1}{d_1}$

$$= 2h_f \sin\frac{\delta_2}{d_2} \tag{11.74}$$

Outside (or tip) diameter of pinion, $d_{a1} = d_1 + 2h_a\cos\delta_1$ (11.75)

Outside diameter of gear, $d_{a2} = d_2 + 2h_a\cos\delta_2$ (11.76)

For right angle gears, $\Sigma = 90°$, and

$$\tan\delta_1 = \frac{z_1}{z_2}$$

$$\tan\delta_2 = \frac{z_2}{z_1}$$

For obtuse angle gears, Σ is greater than 90°, and

$$\tan\delta_2 = \frac{\sin(180° - \Sigma)}{\frac{d_1}{d_2} - \cos(180° - \Sigma)}$$

$$= \frac{\sin(180° - \Sigma)}{\frac{z_1}{z_2} - \cos(180° - \Sigma)} \quad (11.77)$$

Similarly

$$\tan \delta_1 = \frac{\sin(180° - \Sigma)}{\frac{d_2}{d_1} - \cos(180° - \Sigma)}$$

$$= \frac{\sin(180° - \Sigma)}{\frac{z_2}{z_1} - \cos(180° - \Sigma)} \quad (11.78)$$

Forces on straight tooth bevel gears are:

Normal force, $$F_n = \frac{F_t}{\cos \alpha_n} \quad (11.79)$$

Radial force, $$F_r = F_t \tan \alpha_n \sin \delta \quad (11.80)$$

Axial force, $$F_a = F_t \tan \alpha_n \cos \delta \quad (11.81)$$

where $$F_t = \frac{10^3 P}{v_m},\ v_m = \frac{\pi d_m n}{10^3 \times 60} \text{ m/s},$$

and $d_m = d - b \sin \delta$, b = face width of gear tooth.

11.21 SPIRAL GEARS

Spiral gears are used to connect non-parallel and non-intersecting shafts, as shown in Fig.11.27. The shaft angle may be less than or greater than 90°, as shown in figure below.

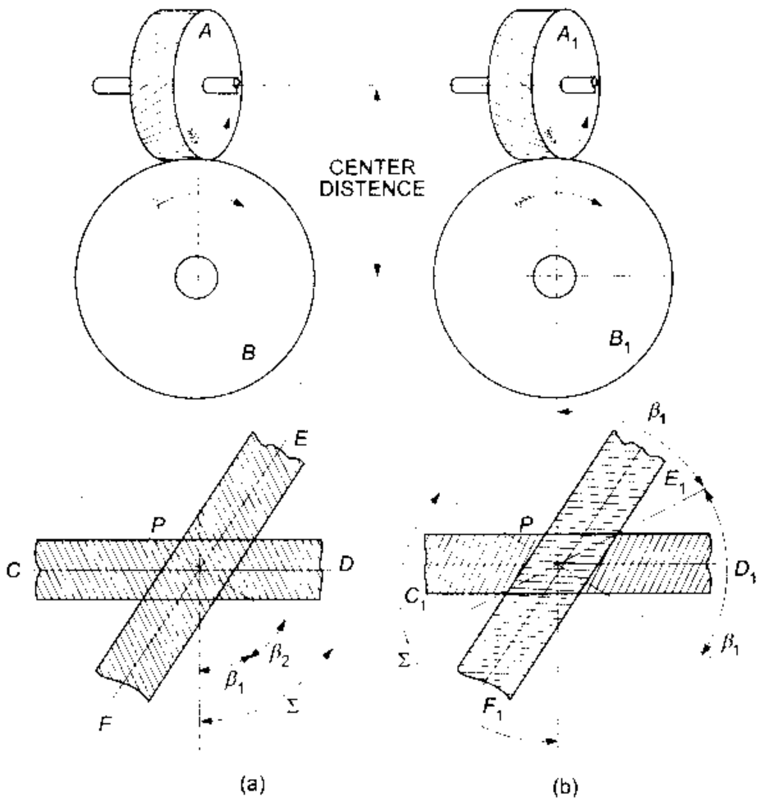


Fig.11.27 Spiral gears

Let β_1 = spiral angle of gear 1

β_2 = spiral angle of gear 2

$\Sigma = \beta_1 + \beta_2$

Gear ratio, $i = \dfrac{z_2}{z_1}$

$$p_1 = \frac{p_n}{\cos\beta_1}, \quad \text{and} \quad p_2 = \frac{p_n}{\cos\beta_2}$$

Also $$d_1 = \frac{z_1 p_1}{\pi} = \frac{z_1 p_n}{\pi \cos\alpha_n}$$

and $$d_2 = \frac{z_2 p_2}{\pi} = \frac{z_2 p_n}{\pi \cos\alpha_n}$$

Centre distance, $$C = \frac{d_1 + d_2}{2}$$

$$= \left(\frac{z_1 p_n}{2\pi}\right)\left[\frac{1}{\cos\beta_1} + \frac{i}{\cos\beta_2}\right] \qquad (11.82)$$

Let $d_v = \frac{z_1 p_n}{\pi}$ be the virtual pitch circle diameter of a spur gear with the same number of teeth and normal pitch as the spiral gear 1.

Thus $$C = \left(\frac{d_v}{2}\right)\left[\frac{1}{\cos\beta_1} + \frac{i}{\cos\beta_2}\right] \qquad (11.83)$$

11.21.1 Efficiency of Spiral Gears

Two spiral gears in mesh at point *P* are shown in Fig.11.28. Gear 1 is the driver and 2 is the driven gear.

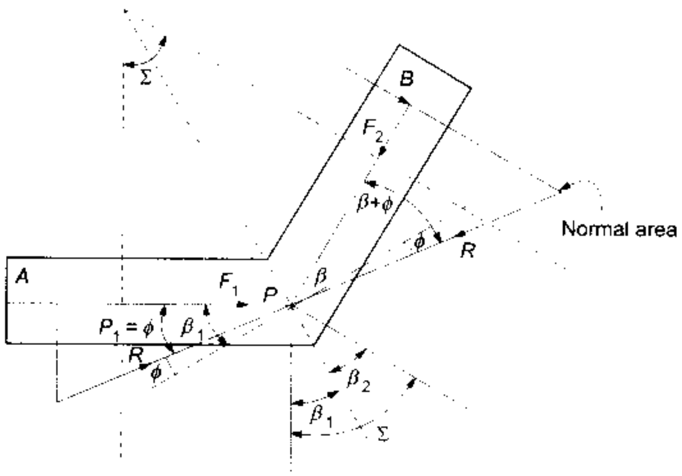


Fig.11.28 Two spiral gears in mesh

Let $\mu = \tan\phi$ be the coefficient of friction between the mating surfaces.

Driving force, $$F_1 = F_2\left[\frac{\cos(\beta_1 - \phi)}{\cos(\beta_2 + \phi)}\right]$$

Without friction, $$F_{10} = \frac{F_2 \cos\beta_1}{\cos\beta_2}$$

Efficiency of the drive,
$$\begin{aligned}
\eta = \frac{F_{10}}{F_1} &= \frac{\cos\beta_1 \cos(\beta_2 + \phi)}{\cos(\beta_1 - \phi)\cos\beta_2} \\
&= \frac{\cos\beta_1 \cos(\Sigma - \beta_1 + \phi)}{\cos(\Sigma - \beta_1)\cos(\beta_1 - \phi)} \\
&= \frac{\cos(\Sigma + \phi) + \cos(2\beta_1 - \Sigma - \phi)}{\cos(\Sigma - \phi) + \cos(\Sigma + \phi - 2\beta_1)} \\
&= \frac{\cos(\Sigma + \phi) + \cos(2\beta_1 - \Sigma - \phi)}{\cos(\Sigma - \phi) + \cos(2\beta_1 - \Sigma - \phi)} \\
&= \frac{\cos(\Sigma + \phi) + \cos(\beta_1 - \beta_2 - \phi)}{\cos(\Sigma - \phi) + \cos(\beta_1 - \beta_2 - \phi)} \qquad (11.84)
\end{aligned}$$

where $\Sigma = \beta_1 + \beta_2$.

For efficiency to be maximum,

$$\cos(2\beta_1 - \Sigma - \phi) = 1$$

or $$2\beta_1 - \Sigma - \phi = 0$$

or $$\beta_1 = \frac{\Sigma + \phi}{2}$$

$$\eta_{max} = \frac{1 + \cos(\Sigma + \phi)}{1 + \cos(\Sigma - \phi)} \qquad (11.85)$$

Example 11.14

Two shafts connected by spiral gears are 500 mm apart. The speed ratio is 3 and the angle between the shafts is 60°. The normal pitch is 20 mm. The spiral angles for the driving and driven gears are equal. Find (a) number of teeth on each gear, (b) exact centre distance and (c) efficiency of the drive. Take friction angle equal to 6°.

■ **Solution**

$$\beta_1 = \beta_2 = \frac{\Sigma}{2} = \frac{60}{2} = 30°$$

Centre distance, $$C = \left(\frac{p_n z_1}{2\pi}\right)\left[\frac{1}{\cos\beta_1} + \frac{i}{\cos\beta_2}\right]$$

$$= \left(\frac{20 z_1}{2\pi}\right)\left[\frac{1}{\cos 30°} + \frac{3}{\cos 30}\right]$$

$$500 = 14.7 z_1$$

or $$z_1 = 34$$

$$z_2 = 34 \times 3 = 102$$

Exact centre distance, $$C = \left(\frac{20 \times 34}{2\pi}\right)\left[\frac{1}{\cos 30°} + \frac{3}{\cos 30°}\right] = 499.87 \text{ mm}$$

Efficiency of the drive,
$$\eta = \frac{\cos(\beta_2 + \phi)\cos\beta_1}{\cos(\beta_1 - \phi)\cos\beta_2}$$
$$= \frac{\cos(30° + 6°)\cos 30°}{\cos(30° - 6°)\cos 30°}$$
$$= 88.56\%$$

Example 11.15

Two spiral gears in mesh have the following data:

Angle of friction = 6°

Normal pitch = 20 mm

Shaft angle = 55°

Speed ratio = 3

Approximate centre distance = 400 mm

Spiral angle of pinion = 25°

Determine (a) the exact centre distance, (b) the number of teeth in each wheel, and (c) the efficiency of the drive.

■ **Solution**

$$C = \left(\frac{z_1 p_n}{2\pi}\right)\left[\frac{1}{\cos\beta_1} + \frac{i}{\cos\beta_2}\right]$$
$$400 = \left(\frac{z_1 \times 20}{2\pi}\right)\left[\frac{1}{\cos 25°} + \frac{3}{\cos 30°}\right]$$
$$z_1 = 27.51 \approx 28$$
$$z_2 = 84$$

Exact centre distance,
$$C = \left(\frac{z_1 p_n}{2\pi}\right)\left[\frac{1}{\cos\beta_1} + \frac{i}{\cos\beta_2}\right]$$
$$= \left(\frac{28 \times 20}{2\pi}\right)\left[\frac{1}{\cos 25°} + \frac{3}{\cos 30°}\right]$$
$$= 407.08 \text{ mm}$$

Efficiency of drive,
$$= \frac{\cos(\beta_2 + \phi)\cos\beta_1}{\cos(\beta_1 - \phi)\cos\beta_2}$$
$$= \frac{\cos(30° + 6°)\cos 25°}{\cos(25° - 6°)\cos 30°}$$
$$= 81.54\%$$

11.22 WORM GEARS

A worm and worm gear is used to provide a high angular velocity reduction between non-intersecting shafts which are usually at right angles. The pinion or worm has a small number of teeth (threads), usually one to four. Its mating gear is called the worm wheel. There is a line contact between the worm threads and the worm wheel teeth. Because of this, worm gears can transmit high tooth loads. However, the high sliding velocities give rise to high heating of the worm. The geometry of a worm and worm gear is shown in Fig.11.29. For a worm and worm gear with shafts at right angles to mesh properly, the following conditions must be satisfied:

1. Lead angle of worm = helix angle of worm gear
2. Axial pitch of worm = circular pitch of worm gear.

Axial diametral pitch (P_x) The axial diametral pitch is the quotient of the number π by the axial pitch.

$$P_x = \frac{\pi}{p_x} \tag{11.86}$$

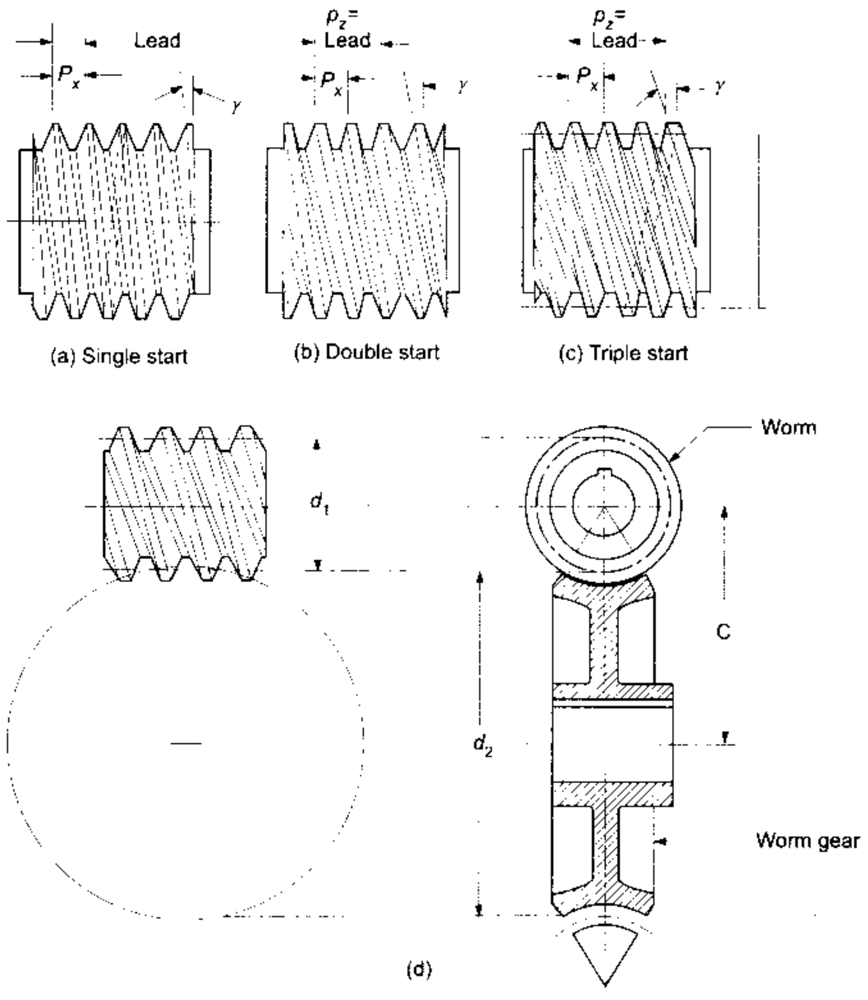


Fig.11.29 Worm gears

Diametral quotient (q) The diametral quotient is the ratio of the reference diameter to the axial module,

$$q = \frac{d}{m} \tag{11.87}$$

Axial module (m_x) The axial module is the quotient of the axial pitch by the number π.

$$m_x = \frac{p_x}{\pi} \tag{11.88}$$

Axial circular pitch (p_x) The axial circular pitch is the distance between two consecutive corresponding profiles, measured parallel to the axis of the worm.

Lead (p_z) Lead is the axial distance between two consecutive intersections of a helix and a straight generator of the cylinder on which it lies.

$$p_z = p_x z_1 \tag{11.89}$$

where z_1 = number of starts on the worm.

Length of the worm The length of the worm is the length of the toothed part of the worm measured parallel to the axis on the reference cylinder.

Gear ratio Gear ratio is the quotient of the number of teeth on the wheel divided by the number of threads on the worm.

$$i = \frac{z_2}{z_1} \tag{11.90}$$

Torus Torus is the surface of revolution generated by the rotation of a circle around an axis external to this circle and situated in its plane.

Gorg Gorg is part of the tip surface in the form of a portion of a torus with the same middle circle diameter as the reference torus.

Tooth width Tooth width is the distance between two planes perpendicular to the axis containing the circles of intersection of the reference torus and the lateral faces of the teeth.

Width angle Width angle is the angle at the centre included between the points of intersection of this circle with the lateral faces of the teeth, in the generating circle of the reference torus.

Lead angle (γ) Lead angle is the angle between a tangent to the pitch helix and the plane of rotation of the worm.

$$\tan\gamma = \frac{p_z}{\pi d_1} = \frac{m_x z_1}{d_1} \tag{11.91}$$

$$\frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{\pi d_2}{p_z} \tag{11.92}$$

$$p_n = p_x \cos\gamma \tag{11.93}$$

$$m_x = \frac{m_n}{\tan\gamma} \tag{11.94}$$

Centre distance, $C = \dfrac{d_1 + d_2}{2} = \left(\dfrac{m_n}{2}\right)\left[\dfrac{z_1}{\cos\beta_1} + \dfrac{z_2}{\cos\beta_2}\right]$

For 90° shafts, $\beta_2 = \gamma$ and $\beta_1 = 90° - \gamma$.

Hence $C = \left(\dfrac{m}{2}\right)\left[z_1 \cot\gamma + z_2\right]$

$$= \left(\frac{p_z}{2\pi}\right)\left[\cos\gamma + i\right] \tag{11.95}$$

where, velocity ratio, $i = \dfrac{z_2}{z_1} = \dfrac{\pi d_2}{p_x} = \dfrac{n_1}{n_2}$

11.22.1 Efficiency of Worm Gears

The efficiency of the worm gears when worm is the driver,

$$\eta = \frac{\cos\beta_1 \cos(\beta_2 + \phi)}{\cos(\beta_1 - \phi)\cos\beta_2}$$

For $\beta_1 + \beta_2 = 90°$; $\beta_2 = \gamma$ and $\beta_1 = 90° - \gamma$, we get

$$\eta = \frac{\sin\gamma \cos(\gamma + \phi)}{\cos\gamma \sin(\gamma + \phi)}$$

$$= \frac{\tan\gamma}{\tan(\gamma + \phi)} \tag{11.96}$$

When worm wheel is the driver, then

$$\eta = \frac{\tan(\gamma - \phi)}{\tan\gamma} \tag{11.97}$$

A worm and worm gear may be considered self-locking when the lead angle of the worm is less than 5°.

Example 11.16

A triple-threaded worm drives a worm gear of 60 teeth, the shaft angle is 90°. The circular pitch of the worm gear is 30 mm and the pitch diameter of the worm is 95 mm. Determine the lead angle of the worm, the helix angle of the worm gear and the distance between shaft centres.

■ **Solution**

Lead,
$$p_z = p_x z_1 = 30 \times 3 = 90 \text{ mm}$$
$$\tan\gamma = \frac{p_z}{\pi d_1} = \frac{90}{\pi \times 95} = 0.30155$$
$$\gamma = 16.78°$$

Helix angle of worm gear = lead angle of worm

Hence
$$\beta_2 = 16.78°$$
$$d_2 = \frac{p_x z_2}{\pi} = \frac{30 \times 60}{\pi} = 572.96 \text{ mm}$$

Centre distance,
$$C = \frac{d_1 + d_2}{2}$$
$$= \frac{95 + 572.96}{2} = 333.98 \text{ mm}$$

Example 11.17

A two start worm rotating at 900 rpm drives a 27 tooth worm gear. The worm has a pitch diameter of 60 mm and a pitch of 20 mm. The coefficient of friction is 0.05. Find (a) the helix angle of worm, (b) the speed of gear, (c) centre distance, (d) efficiency and (e) maximum efficiency.

■ **Solution**

$$N_1 = 900 \text{ rpm}, z_2 = 27, d_1 = 60 \text{ mm}, p_x = 20 \text{ mm}, z_1 = 2$$
$$\phi = \tan^{-1} 0.05 = 2.86°, p_z = p_x z_1 = 20 \times 2 = 40 \text{ mm}$$

(a)
$$\tan\gamma = \frac{p_z}{\pi d_1} = \frac{40}{\pi \times 60} = 0.2122$$
$$\beta_2 = \gamma = 11.98°$$
$$\beta_1 = 90° - \beta_2 = 78.02°$$

(b)
$$d_2 = \frac{p_x z_2}{\pi} = \frac{20 \times 27}{\pi} = 171.88 \text{ mm}$$
$$i = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2} = \frac{\pi d_2}{p_z} = \frac{\pi \times 171.98}{40} = 13.5$$
$$n_2 = \frac{900}{13.5} = 66.67 \text{ rpm}$$

(c)
$$C = \left(\frac{p_z}{2\pi}\right)(\cot\gamma + i) = \left(\frac{40}{2\pi}\right)(\cot 11.98° + 13.5) = 115.95 \text{ mm}$$

(d) $$\eta = \frac{\tan\gamma}{\tan(\gamma+\phi)} = \frac{\tan 11.98°}{\tan(11.98° + 2.86°)}$$
$$= 0.8 \quad \text{or} \quad 80\%$$

(e) $$\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 2.86°}{1+\sin 2.86°} = 90.49\%$$

Exercises

(a) Spur gears

1 The thickness of an involute gear tooth is 8 mm at a radius of 90 mm and a pressure angle of 14.5°. Calculate the tooth thickness and radius at a point on the involute which has a pressure angle of 25°. Also calculate the tooth thickness at the base circle.

2 Two equal spur gears of 48 teeth mesh together with pitch radii of 96 mm and addendum of 4 mm. If the pressure angle is 20°, calculate the length of action and the contact ratio.

3 A pinion with a pitch radius of 40 mm drives a rack. The pressure angle is 20°. Calculate the maximum addendum possible for the rack without having involute interference on the pinion.

4 A 20° pinion having a module of 0.2 and 42 teeth drives a gear of 90 teeth. Calculate the contact ratio.

5 Determine the approximate number of teeth in a 20° involute spur gear such that the base circle diameter will be equal to the dedendum circle diameter.

6 A 20° pinion having a module of 4 and 30 teeth drives a rack. Calculate the length of action and the contact ratio.

7 For a 20° pressure angle, calculate the minimum number of teeth in a pinion to mesh with a rack without involute interference. Also calculate the number of teeth in a pinion to mesh with a gear of equal size without involute interference if the addendum equals the module.

8 A 20° pinion having a module of 2.5 and 40 teeth meshes with a rack with no backlash. If the rack is pulled out 1.25 mm, calculate the backlash error.

9 A 20° pressure angle pinion with a module of 2 and 18 teeth drives a gear of 54 teeth. If the centre distance at which the gear operates is 75 mm, calculate the operating pressure angle, and backlash produced.

10 A pair of meshing spur gears has 22 and 38 teeth, a diametral pitch of 0.32, and a pinion running at 1800 rpm. Determine the following: (a) centre distance, (b) pitch diameter, (c) pitch line velocity, and (d) rpm of the gear.

11 A pair of spur gears has 16 and 18 teeth, a module of 13 mm, addendum of 13 mm, and pressure angle of 14.5°. Show that the gears have interference. Determine the amount by which the addendum must be reduced to eliminate the interference.

12 An internal spur gear having 200 teeth and 20° pressure angle meshes with a pinion having 40 teeth and a module of 2.5 mm. Determine (a) the velocity ratio if the pinion is the driver, (b) the centre distance, and (c) find the resulting pressure angle, if the centre distance is increased by 3 mm.

13 Two spur gears of 24 teeth and 36 teeth of 8 mm module and 20° pressure angle are in mesh. Addendum of each gear is 8 mm. The teeth are of involute form and the pinion rotates at 450 rpm. Determine the velocity of sliding when the pinion is at a radius of 102 mm.

14 A pair of spur gears with involute teeth is to give a gear ratio of 3:1. The arc of approach is not to be less than the circular pitch and the pinion is the driver. The pressure angle is 20°. What is the least number of teeth that can be used on each gear?

15 A pinion with 24 involute teeth of 150 mm pitch circle diameter drives a rack. The addendum of the pinion is 6 mm. Find the least pressure angle which can be used if undercutting of the teeth is to be avoided. Using this pressure angle, find the length of the arc of contact and the minimum number of teeth in contact at one time.

16 A pair of 20° pressure angle gears in mesh has the following data:
Speed of pinion = 400 rpm
Number of teeth on pinion = 24
Number of teeth on gear = 28
Module = 10 mm
Determine the addendum of the gears if the path of approach and recess is half the maximum value. Determine also the arc of contact and the maximum velocity of sliding between the mating surfaces.

17 Two gears in mesh have 10 teeth and 40 teeth respectively. They are full-depth teeth and pressure angle is 20°. The module is 8.5 mm. Determine the (a) reduction in addendum of the gear to avoid interference, and (b) contact ratio.

(b) Helical gears

18 Two standard spur gears are to be replaced by helical gears. The spur gears were cut by a 20° hob having a module of 3, velocity ratio of 1.75:1 and the centre distance is 132 mm. The helical gears are to be cut with the same hob and maintain the same centre distance. The helix angle is to be between 15° and 20° and the velocity ratio between 1.70 and 1.75. Find the number of teeth, the helix angle and the velocity ratio.

19 A pair of helical gears for parallel shafts are to be cut with a hob of module 3. The helix angle is to be 20° and the centre distance between 153 mm and 159 mm. The angular velocity ratio is to approach 2:1 as closely as possible. Calculate the circular pitch and the module in the plane of rotation. Determine the number of teeth, pitch diameters, and the centre distance to satisfy the above conditions.

20 Two crossed shafts are connected by helical gears. The velocity ratio is 18:1 and the shaft angle is 45°. If $d_1 = 60$ mm and $d_2 = 95$ mm, calculate the helix angles if both gears have the same hand.

21 A pair of helical gears having 30 and 48 teeth and a 23° helix angle transmits power between parallel shafts. The module in the normal plane is 3 mm, and the pressure angle in this plane is 20°. Determine (a) the module in the plane of rotation, (b) pitch diameters, (c) the centre distance, (d) the circular pitch in the normal plane, and (e) the circular pitch in the plane of rotation.

22 A pair of crossed helical gears connects shafts making angle of 45°. The right-hand pinion has 36 teeth and a helix angle of 20°. The right-hand gear has 48 teeth and its module in the normal plane is 2.5 mm. Determine (a) The helix angle of the gear, (b) circular pitch in the normal plane, (c) module of the pinion in its plane of rotation, (d) module of the gear in its plane of rotation, and (e) centre distance.

(c) Bevel gears

23 A crown bevel gear of 48 teeth and a module of 2 is driven by a 24-tooth pinion. Calculate the pitch angle of the pinion and the shaft angle.

24 A straight bevel pinion having a module of 6.35 and 14 teeth drives a gear of 20 teeth. The shaft angle is 90°. Calculate the addendum and dedendum, circular tooth thickness for each gear and the pitch and base radii of the equivalent spur gear.

(d) Spiral gears

25 Two shafts inclined at an angle of 65° and with a least distance between them of 175 mm are to be connected by spiral gears of normal circular pitch 15 mm to give a reduction ratio 3:1. Find suitable diameters and number of teeth. Determine also the efficiency of the drive if the spiral angles are determined by the condition of maximum efficiency. The angle of friction is 7°.

26 A spiral reduction gear of ratio 3:2 is to be used on a machine with the angle between the shafts 80°. The approximate centre distance between the shafts is 125 mm. The normal pitch of the teeth is 10 mm and the gear diameters are equal. Find the number of teeth on each gear, pitch circle diameters and spiral angles. Find the efficiency of the drive if the friction angle is 5°.

27 The centre distance between two meshing spiral gears is 200 mm and the angle between the shafts is 60°. The gear ratio is 2 and normal circular pitch 10 mm. The driven gear has a helix angle of 25°. Determine (a) the number of teeth on each wheel, (b) the exact centre distance, and (c) the efficiency if friction angle is 5°.

28 The angle between two shafts is 90°. They are joined by two spiral gears having a normal circular pitch of 8 mm and gear ratio of 3. If the approximate centre distance between the shafts is 250 mm and friction angle 6°, determine for the maximum efficiency of the drive (a) the number of teeth, (b) the exact centre distance, (c) pitch diameters, and (d) the efficiency.

(e) Worm gears

29 A double-threaded worm having a lead of 65 mm drives a worm gear with a velocity ratio of 20:1; the angle between the shafts is 90°. If the centre distance is 235 mm, determine the pitch diameter of the worm and worm gear.

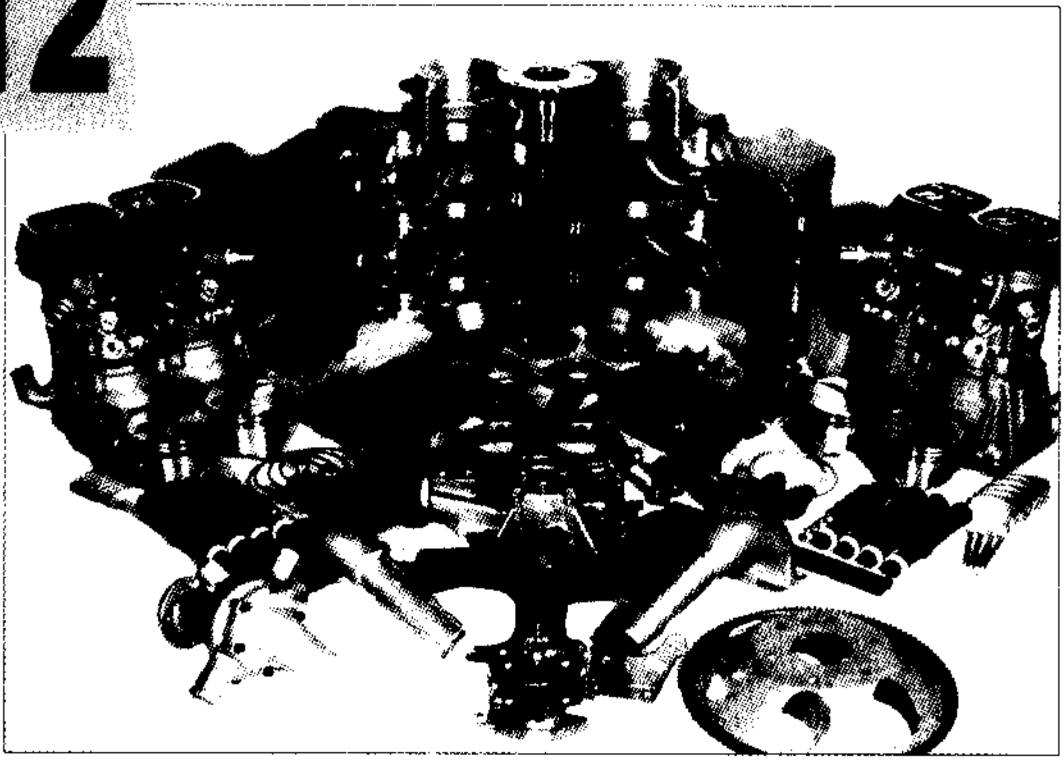
30 A worm and worm gear with shafts at 90° and a centre distance of 178 mm are to have a velocity ratio of 18:1. If the axial pitch of the worm is to be 26.192 mm, determine the maximum number of teeth in the worm and worm gear that can be used for the drive and their corresponding pitch diameters.

31 A double-threaded worm drives a 31-tooth worm gear with shafts at 90°. If the centre distance is 210 mm and the lead angle of the worm 18.83°, calculate the axial pitch of the worm and the pitch diameters of the two gears.

32 A worm and worm gear with shafts at 90° and a centre distance of 76 mm are to have a velocity ratio of 8:1. Using a lead angle of 28.88°, determine the pitch diameters. Select number of teeth for the gears considering worms with 1 to 10 threads.

33 A worm and worm gear have axes at 90° and give a speed reduction of 15 to 1. The triple-thread worm has a lead angle of 20° and an axial pitch of 10 mm. Determine the following for the worm gear: (a) the number of teeth, (b) the pitch diameter, and (c) the helix angle.

12



GEAR TRAINS

12.1 INTRODUCTION

A gear train is composed of two or more gears in mesh for the purpose of transmitting motion from one shaft to another. A gear train enables to have larger centre distance between the driving and driven shafts, provides control on the direction of rotation of the driven gear and facilitates increased transmission ratio. In this chapter, we shall study the various types of gear trains.

12.2 TYPES OF GEAR TRAINS

There are four types of gear trains:

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Planetary (or epicyclic) gear train.

12.2.1 Simple Gear Train

A simple gear train is one in which there is only one gear on each shaft. A simple gear train is often used to change the direction of rotation of a gear without changing its angular velocity. This can be done by placing an idler gear between the driving and driven gears.

Consider the simple gear train shown in Fig.12.1.

$$\frac{n_1}{n_2} = \frac{z_2}{z_1}$$

$$\frac{n_2}{n_3} = \frac{z_3}{z_2}$$

$$\vdots \qquad \vdots$$

$$\frac{n_{m-1}}{n_m} = \frac{z_m}{z_{m-1}}$$

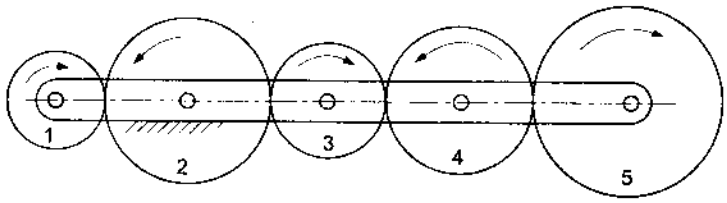


Fig.12.1 Simple gear train

Multiplying, we get

$$\frac{n_1}{n_m} = \frac{z_m}{z_1} \tag{12.1}$$

Therefore, the velocity ratio of a simple gear train is the ratio of the angular velocity of the first gear in the train to the angular velocity of the last gear. We find that the intermediate gears do not in any way affect the velocity ratio. These gears are called *idler gears*. If the number of gears in the train are even then the direction of rotation of the last gear is reversed and if the number of gears in the train are odd then the direction of rotation of the last gear remains the same. Idler gears are used for two purposes: to connect gears where a large centre distance is required and to control the directional relationship between gears.

12.2.2 Compound Gear Train

A pair of gears is compound if they have a common axis and are integral. A compound gear train is a gear train containing compound gears. Consider the compound gear train shown in Fig.12.2. Gears 2 and 3 are on the same shaft.

$$\frac{n_2}{n_1} = -\frac{z_1}{z_2}$$

$$\frac{n_4}{n_3} = -\frac{z_3}{z_4}$$

Multiplying, we get

$$\frac{n_2}{n_1} \times \frac{n_4}{n_3} = \frac{z_1}{z_2} \times \frac{z_3}{z_4}$$

But

$$n_2 = n_3$$

Thus

$$\frac{n_4}{n_1} = \frac{z_1}{z_2} \times \frac{z_3}{z_4} = \frac{z_1 z_3}{z_2 z_4}$$

$$\frac{\text{Speed of driven gear}}{\text{Speed of driving gear}} = \frac{\text{Product of teeth of driving gears}}{\text{Product of teeth of driven gears}} \tag{12.2}$$

The ratio given by (12.2) is called the train value.

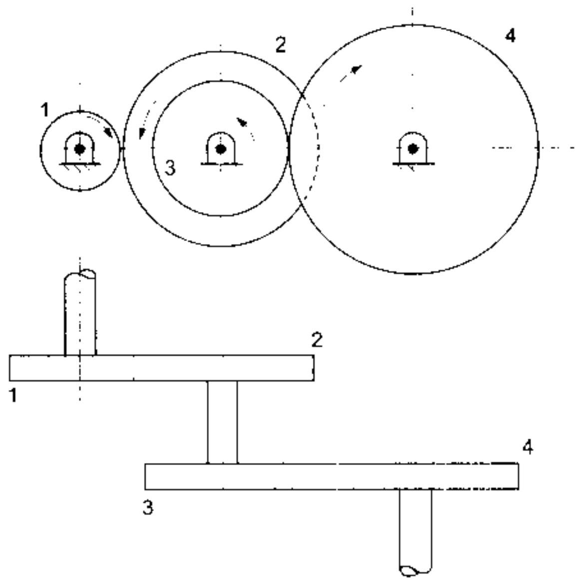


Fig.12.2 Compound gear train

12.2.3 Reverted Gear Train

In a reverted gear train, the first and the last gear are coaxial. Reverted gear trains are used in automotive transmission, lathe back gears, industrial speed reducers, and in clocks where the minute and hour hand shafts are coaxial. In the reverted gear train shown in Fig.12.3, we have

Centre distance, $C = r_1 + r_2 = r_3 + r_4$

For same module $\left(m = \frac{d}{z}\right)$ of all gears, the pitch radius of gears is proportional to the number of teeth. Hence

$$z_1 + z_2 = z_3 + z_4 \qquad (12.3)$$

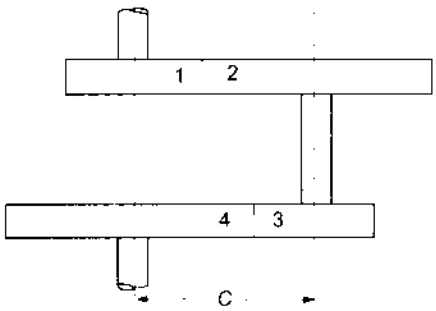


Fig.12.3 Reverted gear train

12.2.4 Planetary Gear Trains

These are gear trains in which the axis of one or more gears move relative to the frame. The gear at the centre is called the sun and the gears whose axes move are called the planets.

In Fig.12.4, arm 3 drives gear 1 about gear 2, which is a fixed external gear. Gear 1 rotates about its centre O_1 while its centre rotates about centre O_2 of the fixed gear. As gear 1 rolls on the outside of gear 2, a point on its surface will generate an epicycloid. If gear 2 happens to be an internal gear and gear 1 rolls on the inside of gear 2, then a point on the surface of gear 1 will generate a hypocycloid. Because of the curves generated, a planetary gear train is often called as an epicyclic, or cyclic, gear train.

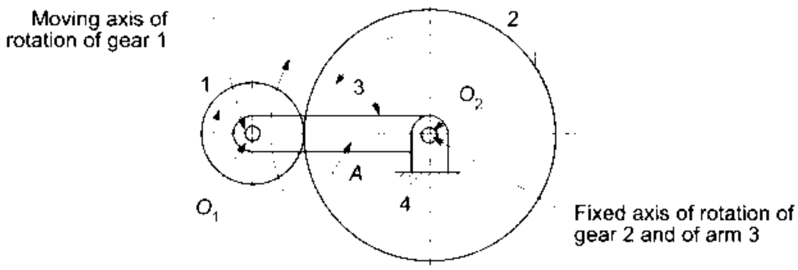


Fig.12.4 Planetary gear train

The speed ratio of a planetary gear train can be determined by the following methods:

1. Relative velocity method
2. Algebraic or tabular method

Relative velocity method Consider the planetary gear train shown in Fig.12.4.

$$\omega_{13} = \omega_{14} - \omega_{34}$$

$$\omega_{23} = \omega_{24} - \omega_{34}$$

and
$$\frac{\omega_{13}}{\omega_{23}} = \frac{\omega_{14} - \omega_{34}}{\omega_{24} - \omega_{34}}$$

Link 3 is the arm, link 1 is the driven gear, link 2 is the driving gear, and link 4 is fixed. If subscript 'a' is used for the arm, then

$$\frac{\omega_{1a}}{\omega_{2a}} = \frac{\omega_1 - \omega_a}{\omega_2 - \omega_a}$$

or
$$\frac{n_{1a}}{n_{2a}} = \frac{n_1 - n_a}{n_2 - n_a}$$

Now
$$\frac{n_{1a}}{n_{2a}} = -\frac{z_1}{z_2}$$

Hence,
$$\frac{n_1 - n_a}{n_2 - n_a} = -\frac{z_1}{z_2}$$

If gear 2 is fixed, then $n_2 = 0$. Thus

$$\frac{n_1}{n_a} = \left(1 + \frac{z_1}{z_2}\right) \tag{12.4}$$

If arm is fixed, then $n_a = 0$. Thus

$$\frac{n_1}{n_2} = -\frac{z_1}{z_2} \tag{12.5}$$

Tabular method This method is based on the principle of superposition, which states that the resultant revolutions or turns of any gear may be found by taking the number of turns it makes with the arm plus the number of turns it makes relative to the arm. The following steps may be followed for this method:

1. Assume the arm to be fixed and determine the revolutions of different gears of the train for one revolution of a particular convenient gear.
2. Multiply all columns in the first row by x and write in the second row.
3. To account for rotation of arm, add y to the various quantities of second row.
4. Out of the three quantities involved in the last row, two of them are given. From these the values of x and y can be determined. On substituting in the third, its magnitude can be determined.

Table 12.1 may be conveniently used for this purpose:

Table 12.1

Operation	Revolutions of		
	Arm *A*	Gear 1	Gear 2
1. Arm *A* fixed +1 revolutions given to gear 1.	0	+1	$\frac{-z_1}{z_2}$
2. Multiply by x	0	$+x$	$\frac{-xz_1}{z_2}$
3. Add y	$+y$	$x+y$	$\frac{-xz_1}{z_2}+y$

12.3 SUN AND PLANET GEARS

Figure 12.5 shows the sun and planet gears in which gear *P* is the planet, gear *S* the sun, arm *A*, and the annular (or internal) gear 1. The annular gear is fixed and the planet gear rolls over the sun and the annular gear. O_1 is the moving axis of rotation of planet gear *P* and O_2 is the fixed axis of rotation of sun gear 2 and arm *A*. The axes O_1 and O_2 are coupled by the arm *A*. Let z_p, z_s and z_i be the number of teeth on the planet, sun and annular gears respectively. To find the speed of any gear, Table 12.2 may be used:

Table 12.2

Operation	Arm A	Sun gear S	Planet gear P	Annular gear I
1. Arm fixed	0	+1	$\frac{-z_s}{z_p}$	$\frac{-z_s}{z_i}$
2. Multiply by x	0	$+x$	$\frac{-z_s}{z_p}\cdot x$	$\frac{-z_s}{z_i}\cdot x$
3. Add y	y	$x+y$	$-\frac{z_s}{z_p}\cdot x+y$	$-\frac{z_s}{z_i}\cdot x+y$

Let n_s and n_i be the speed of sun and annular gears respectively.

Then $$x+y=n_s$$

$$-\left(\frac{z_s}{z_i}\right)x + y = n_i$$

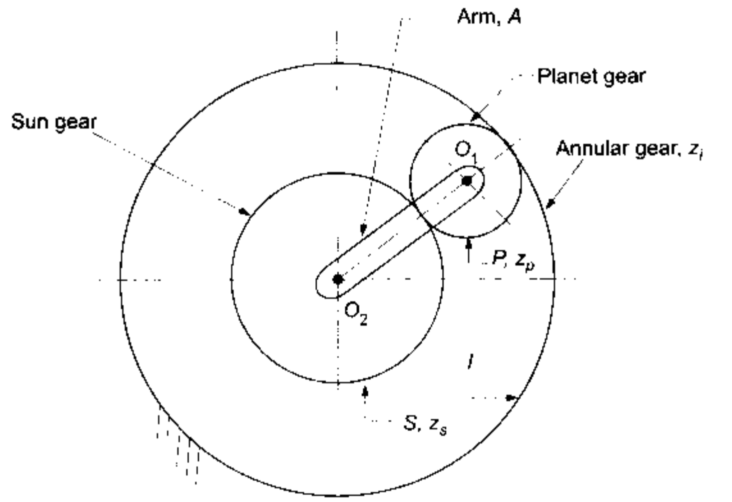


Fig.12.5 Sun and planet gears

Solving for x and y, we get

$$x = \frac{z_i n_s - z_i n_i}{z_i + z_s}$$

$$y = \frac{z_s n_s + z_i n_i}{z_i + z_s}$$

(a) When sun gear is fixed, then $n_s = 0$. Thus

$$y = \frac{-z_i n_i}{z_i + z_s} = \frac{-n_i}{1 + z_s/z_i} \tag{12.6}$$

(b) When annular gear is fixed, then $n_i = 0$. Thus

$$y = \frac{z_i n_s}{z_i + z_s} = \frac{n_s}{1 + \frac{z_i}{z_s}} \tag{12.7}$$

12.4 EPICYCLICS WITH TWO INPUTS

A gear train of this type is shown in Fig.12.6. Let n_1, n_2 and n_o represent the turns of input 1, input 2 and the output, respectively. By superposition, the number of turns of the output equals the output turns due to input 1 plus the output turns due to input 2. This can be expressed as

$$n_o = n_1 \left(\frac{n_o}{n_1}\right)_{\text{input 2 held fixed}} + n_2 \left(\frac{n_o}{n_2}\right)_{\text{input 1 held fixed}} \tag{12.8}$$

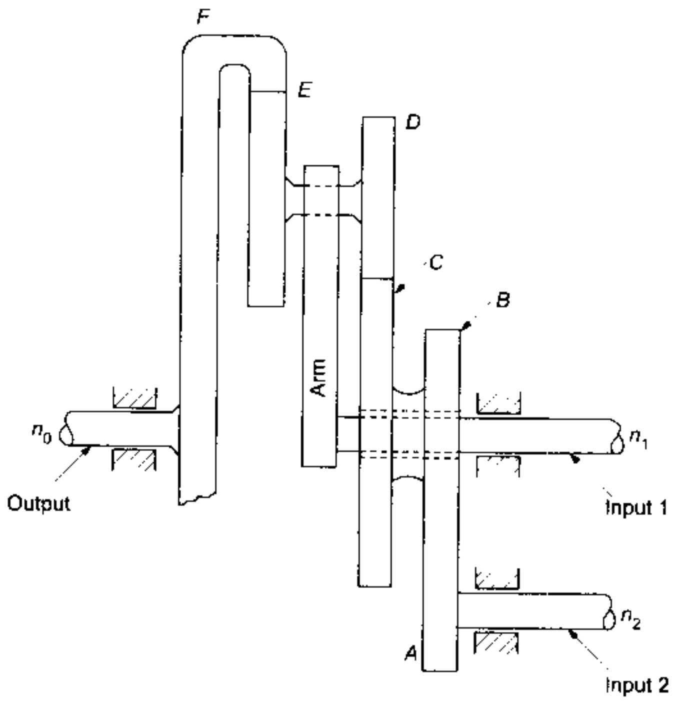


Fig.12.6 Epicyclics with two inputs

12.5 COMPOUND EPICYCLIC GEAR TRAIN

A compound epicyclic gear train consists of two or more epicyclic gear trains connected in series. A compound epicyclic gear train is analysed by considering each epicyclic gear train separately. That epicyclic gear train is analysed first where two conditions (or speeds of two elements) are known.

12.6 EPICYCLIC BEVEL GEAR TRAINS

Bevel gears can be used to make a more compact epicyclic system and they permit a very high speed reduction with few gears. They find potential applications in speed reduction gears and differential gear of an automobile.

12.7 TORQUES IN EPICYCLIC GEAR TRAINS

Consider the rotating parts of an epicyclic gear train shown in Fig.12.7. If the rotating parts have no angular acceleration, then the gear train is kept in equilibrium by the following three externally applied torques:

1. Input torque (T_1) on the driving member
2. Output torque (T_2) on the driven member
3. Braking torque (T_3) on the fixed member

The net torque applied on the gear train must be zero, that is

$$T_1 + T_2 + T_3 = 0$$

If F_1, F_2 and F_3 are the externally applied forces at radii r_1, r_2 and r_3, then

$$F_1 r_1 + F_2 r_2 + F_3 r_3 = 0$$

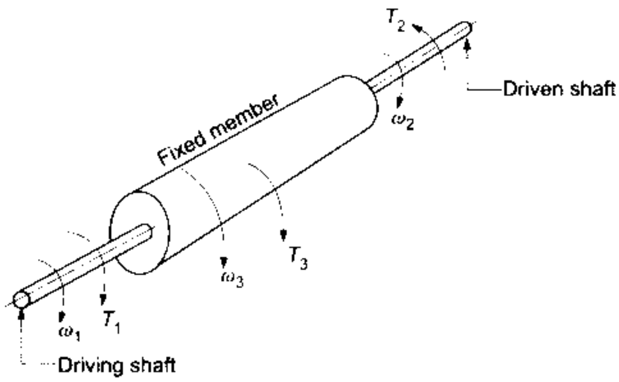


Fig.12.7 Torque in epicyclic gear trains

If ω_1, ω_2, and ω_3 are the angular velocities of the driving, driven, and fixed members respectively, and friction is neglected, then the net kinetic energy dissipated by the gear train must be zero.

$$T_1\omega_1 + T_2\omega_2 + T_3\omega_3 = 0$$

For a fixed member, $\omega_3 = 0$. Therefore

$$\begin{aligned} T_1\omega_1 + T_2\omega_2 &= 0 \\ T_2 &= -\left(\frac{\omega_1}{\omega_2}\right)T_1 \\ T_3 &= -(T_1 + T_2) \\ &= T_1\left(\frac{\omega_1}{\omega_2} - 1\right) \\ &= T_1\left(\frac{n_1}{n_2} - 1\right) \qquad (12.9) \end{aligned}$$

Example 12.1

The speed ratio of a reverted gear train is 15. The module of gears 1 and 2 is 3 mm and that of gears 3 and 4 is 2.5 mm. Calculate the suitable number of teeth for the gears. The centre distance between gear shafts is 250 mm.

■ Solution

$$\frac{n_1}{n_2} = \frac{n_3}{n_4} = \sqrt{15} = 3.873$$

or

$$\frac{z_2}{z_1} = \frac{z_4}{z_3} = 3.873$$

Now

$$r_1 + r_2 = r_3 + r_4 = 250 \text{ mm}$$

or

$$\frac{m_1(z_1 + z_2)}{2} = \frac{m_2(z_3 + z_4)}{2} = 250$$

or

$$3(z_1 + z_2) = 2.5(z_3 + z_4) = 500$$

$$z_1 + z_2 = \frac{500}{3}$$

$$\text{or} \quad 4.873z_1 = \frac{500}{3}$$

$$\text{or} \quad z_1 = 34.2 \approx 34$$

$$z_2 = 133$$

$$z_3 + z_4 = 200$$

$$\text{or} \quad 4.873z_3 = 200$$

$$\text{or} \quad z_3 = 41$$

$$z_4 = 160$$

Example 12.2

In an epicyclic gear train, an arm carries two gears 1 and 2 having 40 and 50 teeth, respectively. The arm rotates at 160 rpm counter-clockwise about the centre of gear 1, which is fixed. Determine the speed of the gear 2.

■ Solution

Table 12.3 can be used to find the speed of the gear.

Table 12.3

Operation	Revolutions of		
	Arm A	Gear 1	Gear 2
1. Arm A fixed	0	+1	$\frac{-z_1}{z_2}$
2. Multiply by x	0	$+x$	$\frac{-xz_1}{z_2}$
3. Add y	$+y$	$x+y$	$\frac{-xz_1}{z_2}+y$

As year 1 is fixed,

$$x + y = 0$$

$$\text{or} \quad x = -y = -160 \text{ rpm}$$

Speed of gear 2,

$$n_2 = \frac{-xz_1}{z_2} + y$$

$$= 160\left(\frac{40}{50}\right) + 160$$

$$= 288 \text{ rpm ccw}$$

Example 12.3

In a reverted epicyclic gear train, the arm A carries two gears 1 and 2 at centre of rotation O_1 and a compound gear 3 and 4 at centre of rotation O_2. The gear 1 meshes with gear 4 and the gear 2 meshes with gear 3. The number of teeth are $z_1 = 75$, $z_2 = 30$ and $z_3 = 90$. Find the speed and direction of gear 2 when gear 1 is fixed and the arm A makes 120 rpm clockwise. Assume all gears to be of the same module.

■ Solution

$$r_1 + r_4 = r_2 + r_3$$

$$\text{or} \quad z_1 + z_4 = z_2 + z_3$$

$$z_4 = 30 + 90 - 75 = 45$$

Table 12.4 can be used to find the speed of the gears.

Table 12.4

Operation	Revolutions of			
	Arm *A*	Compound gear 3,4	Gear 1	Gear 2
1. Arm *A* fixed	0	+1	$\frac{-z_4}{z_1}$	$\frac{-z_3}{z_2}$
2. Multiply by x	0	$+x$	$\frac{-xz_4}{z_1}$	$\frac{-xz_3}{z_2}$
3. Add y	$+y$	$x+y$	$\frac{-xz_4}{z_1}+y$	$\frac{-xz_3}{z_2}+y$

Since gear 1 is fixed,

$$\frac{-xz_4}{z_1} + y = 0$$

or

$$-x\left(\frac{45}{75}\right) + y = 0$$

or

$$-0.6x + y = 0 \qquad (1)$$

Arm *A* makes 120 rpm clockwise, therefore

$$y = -120 \qquad (2)$$

From (1) and (2), we get

$$x = -200 \text{ rpm}$$

Now for gear 2, we have

$$n_2 = \frac{-xz_3}{z_2} + y$$

$$= 200\left(\frac{90}{30}\right) - 120$$

$$= 480 \text{ rpm ccw}$$

Example 12.4

An epicyclic gear train consists of three gears 1, 2 and 3 as shown in Fig.12.8. The internal gear 1 has 72 teeth and gear 3 has 32 teeth. The gear 2 meshes with both gear 1 and gear 3 and is carried on an arm *A* which rotates about the centre O_2 at 20 rpm. If the gear 1 is fixed, determine the speed of gears 2 and 3.

■ Solution

Table 12.5 can be used to find the speed fo the gears.

Table 12.5

Operation	Revolutions of			
	Arm *A*	Gear 3	Gear 1	Gear 2
1. Arm *A* fixed	0	+1	$\frac{-z_3}{z_1}$	$\frac{-z_3}{z_2}$
2. Multiply by x	0	$+x$	$\frac{-xz_3}{z_1}$	$\frac{-xz_3}{z_2}$
3. Add y	$+y$	$x+y$	$\frac{-xz_3}{z_1}+y$	$\frac{-xz_3}{z_2}+y$

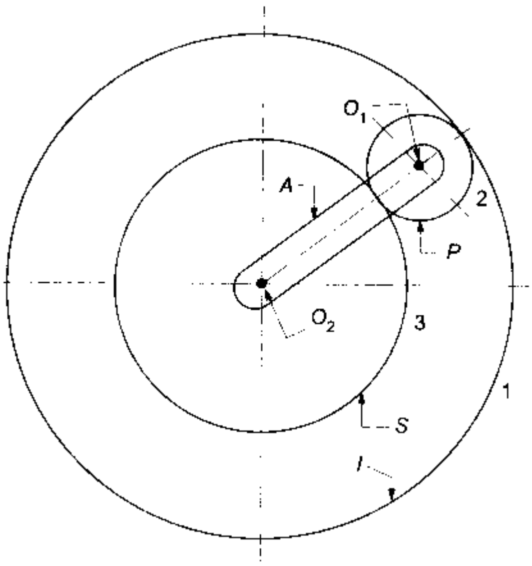


Fig.12.8 Epicyclic gear train

Speed of arm,

$$y = 20 \text{ rpm}$$

For gear 1 fixed, we have

$$\frac{-xz_3}{z_2 + y} = 0$$

$$-x\left(\frac{32}{72}\right) + 20 = 0$$

$$x = 45$$

Speed of gear 3

$$n_3 = x + y$$
$$= 45 + 20 = 65 \text{ rpm in the direction of arm}$$

Speed of gear 2

$$d_2 + \frac{d_3}{2} = \frac{d_1}{2}$$

or
$$2d_2 + d_3 = d_1$$

or
$$2z_2 + z_3 = z_1$$

or
$$2z_2 + 32 = 72$$

$$z_2 = 20$$

$$n_2 = -x\left(\frac{z_3}{z_2}\right) + y$$

$$= -45\left(\frac{32}{20}\right) + 20$$

$$= -52 \text{ rpm in the opposite direction of arm.}$$

Example 12.5

The pitch circle diameter of the annular gear in the epicyclic gear train shown in Fig.12.9 is 425 mm and the module is 5 mm. When the annular gear 3 is stationary, the spindle *A* makes one revolution in the same sense as the sun gear 1 for every 6 revolutions of the driving spindle carrying the sun gear. All the planet gears are of the same size. Determine the number of teeth on all the gears.

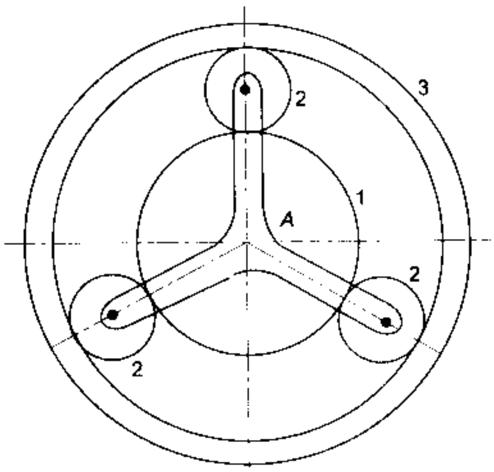


Fig.12.9 Epicyclic gear train

Solution

Table 12.6 can be used to find the speed of the gears.

Table 12.6

Operation	Revolutions of			
	Spindle *A*	Gear 1	Gear 2	Gear 3
1. Arm *A* fixed	0	+1	$\frac{-z_1}{z_2}$	$\frac{-z_1}{z_3}$
2. Multiply by x	0	$+x$	$-\frac{xz_1}{z_2}$	$-\frac{xz_1}{z_3}$
3. Add y	$+y$	$x+y$	$\frac{-xz_1}{z_2}+y$	$\frac{-xz_1}{z_3}+y$

Now $\quad y = +1$

and $\quad x + y = 6$

Therefore, $\quad x = 5$

Gear 3 is stationary, hence

$$\frac{-xz_1}{z_3} + y = 0$$

$$\frac{-5z_1}{z_3} + 1 = 0$$

$$\frac{z_1}{z_3} = \frac{1}{5}$$

Now $$z_3 = \frac{d_3}{m} = \frac{425}{5} = 85$$

$$z_1 = \frac{85}{5} = 17$$

Also $$d_1 + 2d_2 = d_3$$

or $$z_1 + 2z_2 = z_3$$

$$17 + 2z_2 = 85$$

$$z_2 = 34$$

Example 12.8

The Ferguson's paradox epicyclic gear train is shown in Fig.12.10. Gear 1 is fixed to the frame. The arm *A* and gears 2 and 3 are free to rotate on the shaft *S*. Gears 1, 2, and 3 have 100, 101 and 99 teeth respectively. The planet gear has 20 teeth. The pitch circle diameter of all the gears is the same so that the planet gear *P* meshes with all of them. Determine the revolutions of gears 2 and 3 for one revolution of the arm *A*.

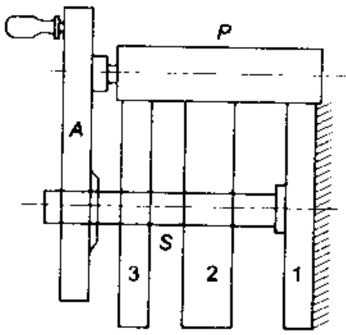


Fig.12.10 Ferguson's paradox epicyclic gear train

■ Solution

Table 12.7 can be used to find the speed of the gears.

Table 12.7

Operation	Revolutions of			
	Spindle *A*	Gear 1	Gear 2	Gear 3
1. Arm *A* fixed	0	+1	$\frac{z_1}{z_2}$	$\frac{z_1}{z_3}$
2. Multiply by x	0	$+x$	$\frac{xz_1}{z_2}$	$\frac{xz_1}{z_3}$
3. Add y	$+y$	$x + y$	$\frac{xz_1}{z_2} + y$	$\frac{xz_1}{z_3} + y$

Now

$$y = 1$$

Gear 1 is fixed, therefore

$$x + y = 0$$

or

$$x = -y = -1$$

Also

$$n_2 = \frac{xz_1}{z_2} + y = \frac{-100}{101} + 1 = \frac{1}{101}$$

$$n_3 = \frac{xz_1}{z_3} + y = \frac{-100}{99} + 1 = \frac{-1}{99}$$

Example 12.7

In the gear drive shown in Fig.12.11, the driving shaft *A* rotates at 300 rpm in the clockwise direction, when seen from the left hand side. The shaft *B* is the driven shaft. The casing *C* is held stationary. The wheels *E* and *H* are keyed to the central vertical spindle and wheel *F* can rotate freely on this spindle. The wheels *K* and *L* are rigidly fixed to each other and rotate together freely on a pin fitted on the underside of *F*. The wheel *L* meshes with internal teeth on the casing *C*. The number of teeth on the different gears are indicated within brackets.

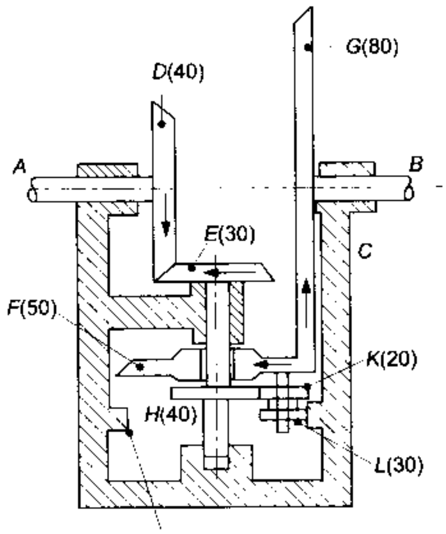


Fig.12.11 Gear drive

Determine the number of teeth on gear *C* and the speed and direction of rotation of shaft *B*.

■ Solution

The wheels *D* and *G* are auxiliary gears and do not form a part of the epicyclic gear train.

$$n_E = n_A \frac{z_D}{z_E} = 300\left(\frac{40}{30}\right) = 400 \text{ rpm cw}$$

Assuming same module for all the gears,

$$z_C = z_H + z_K + z_L = 40 + 20 + 30 = 90$$

Table 12.8 can be used to find the speed of the gears.

Table 12.8

Operation	Revolutions of			
	Wheel *F*	Gears *E* & *H*	Gear *K* & *L*	Gears *C*
1. Arm *A* fixed	0	-1	$\frac{z_H}{z_K}$	$\left(\frac{z_H}{z_K}\right)\left(\frac{z_L}{z_C}\right)$
2. Multiply by x	0	$-x$	$\frac{xz_H}{z_K}$	$x\left(\frac{z_H}{z_K}\right)\left(\frac{z_L}{z_C}\right)$
3. Add $-y$	$-y$	$-x-y$	$\frac{xz_H}{z_K} - y$	$x\left(\frac{z_H}{z_K}\right)\left(\frac{z_L}{z_C}\right) - y$

$$-x - y = -400$$

or $$x + y = 400 \quad (1)$$

Now wheel *C* is fixed, therefore

$$x\left(\frac{z_H}{z_K}\right)\left(\frac{z_L}{z_C}\right) - y = 0$$

or $$x\left(\frac{40}{20}\right)\left(\frac{30}{90}\right) - y = 0$$

or $$\frac{2x}{3} - y = 0 \quad (2)$$

From (1) and (2), we have

$$x = 240 \quad \text{and} \quad y = 160$$

Thus $$n_F = -y = -160 \text{ rpm}$$

Speed of wheel *G* or shaft, $$B = -n_F \frac{z_F}{z_G}$$

$$= -\left(-160 \times \frac{50}{80}\right) = 100 \text{ rpm ccw}$$

Example 12.8

In a gear train, as shown in Fig. 12.12, gear *B* is connected to the input shaft. The arm *A* carrying the compound wheels *D* and *E*, turns freely on the output shaft. If the input speed is 1200 rpm counter-clockwise, when seen from the right, determine the speed of the output shaft under the following conditions: (a) when the gear *C* is fixed and (b) when gear *C* rotates at 10 rpm counter-clockwise.

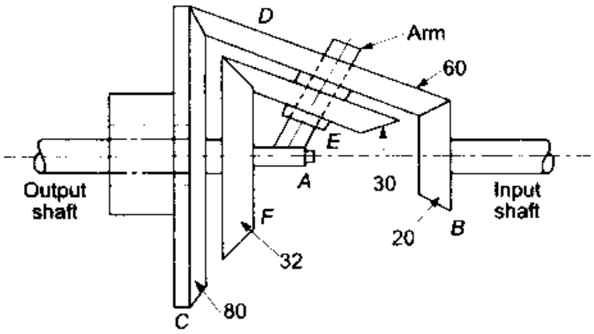


Fig.12.12 Gear train

■ Solution

Table 12.9 can be used to find the speed of the gears.

Table 12.9

Operation	Revolutions of				
	Arm *A*	Gear *B* (input shaft)	Compound gears *D*, *E*	Gear *C*	Gear *F* (output shaft)
1. Arm *A* fixed	0	1	$\frac{z_B}{z_D}$	$\frac{-z_B}{z_C}$	$-\left(\frac{z_B}{z_D}\right)\left(\frac{z_E}{z_F}\right)$
2. Multiply by x	0	x	$\frac{xz_B}{z_D}$	$\frac{-xz_B}{z_C}$	$-x\left(\frac{z_B}{z_D}\right)\left(\frac{z_E}{z_F}\right)$
3. Add y	y	$x+y$	$\frac{xz_B}{z_D}+y$	$\frac{-xz_B}{z_C}+y$	$-x\left(\frac{z_B}{z_D}\right)\left(\frac{z_E}{z_F}\right)+y$

(a) Gear *C* is fixed, therefore

$$-x\left(\frac{z_B}{z_C}\right)+y = 0$$

$$-x\left(\frac{20}{80}\right)+y = 0$$

or $$-0.25x+y = 0 \quad (1)$$

Now $$x+y = +1200 \quad (2)$$

From (1) and (2), we get

$$x = +960 \quad \text{and} \quad y = +240$$

$$n_F = -x\left(\frac{z_B}{z_D}\right)\left(\frac{z_E}{z_F}\right)+y$$

$$= -960\left(\frac{20}{80}\right)\left(\frac{30}{32}\right)+240$$

$$= 15 \text{ rpm (counter-clockwise)}$$

(b) When gear C is rotating at 10 rpm clockwise, we have

$$-x\left(\frac{z_B}{z_C}\right) + y = +10$$

or

$$-x\left(\frac{20}{80}\right) + y = 10$$

or

$$-0.25x + y = 10 \quad (3)$$

From (2) and (3), we get

$$x = 952 \quad \text{and} \quad y = 248$$

$$n_F = -x\left(\frac{z_B}{z_D}\right)\left(\frac{z_E}{z_F}\right) + y$$

$$= -952\left(\frac{20}{80}\right)\left(\frac{30}{32}\right) + 248$$

$$= 24.875 \text{ rpm (counter-clockwise)}$$

Example 12.9

The differential gear used in an automobile is shown in Fig.12.13. The pinion A on the propeller shaft has 12 teeth and the crown gear B has 60 teeth. The shafts P and Q form the rear axles to which the road wheels are attached. If the propeller shaft rotates at 1200 rpm and the road wheel attached to axle Q has a speed of 250 rpm while taking a turn, find the speed of road wheel attached to axle P.

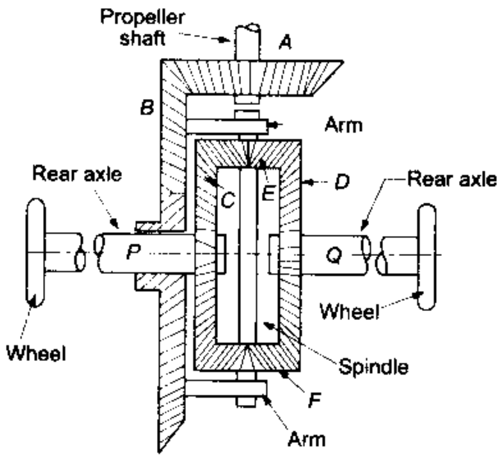


Fig.12.13 Differential gear

■ **Solution**

$$n_B = \left(\frac{z_A}{z_B}\right) n_A = \left(\frac{12}{60}\right) 1200 = 240 \text{ rpm}$$

Table 12.10 can be used to find the speed of the gears.

Table 12.10

Operation	Revolutions of			
	Gear B	Gear C	Gear E	Gear D
1. Gear B fixed	0	1	$\frac{z_C}{z_E}$	$-\left(\frac{z_C}{z_E}\right)\left(\frac{z_E}{z_D}\right) = -1$ or $\frac{z_C}{z_D} = 1$
2. Multiply by x	0	x	$x\left(\frac{z_C}{z_E}\right)$	$-x$
3. Add y	y	$x + y$	$x\frac{z_C}{z_E} + y$	$-x + y$

Now $\quad y = 240$ rpm

Also $\quad -x + y = 250$

or $\quad x = 240 - 250 = -10$

speed of road wheel attached to axle P = speed of gear C

$$= x + y$$
$$= -10 + 240 = 230 \text{ rpm}$$

Example 12.10

An epicyclic gear train consists of a sun wheel S, a stationary annular wheel E and three identical planet wheels P carried on a star-shaped carrier C, as shown in Fig.12.14. The size of different toothed wheels is such that the planet carrier C rotates at 1/5 th of the speed of the sun wheel S. The minimum number of teeth on any wheel is 18. The driving torque on the sun wheel is 120 Nm. Determine (a) the number of teeth on different wheels of the train, and (b) the torque necessary to keep the internal gear stationary.

■ **Solution**

(a) $\quad n_C = n_S/5$

Table 12.11 can be used to find the speed of the gears.

Table 12.11

Operation	Revolutions of			
	Planet carrier C	Sun wheel S	Planet wheel P	Annular wheel E
1. Planet C fixed	0	1	$\frac{-z_S}{z_P}$	$-\left(\frac{z_S}{z_P}\right)\left(\frac{z_P}{z_E}\right) = \frac{-z_S}{z_E}$
2. Multiply by x	0	x	$-x\frac{z_S}{z_P}$	$-x\frac{z_S}{z_E}$
3. Add y	y	$x + y$	$-x\frac{z_S}{z_P} + y$	$y - \frac{x z_S}{z_E}$

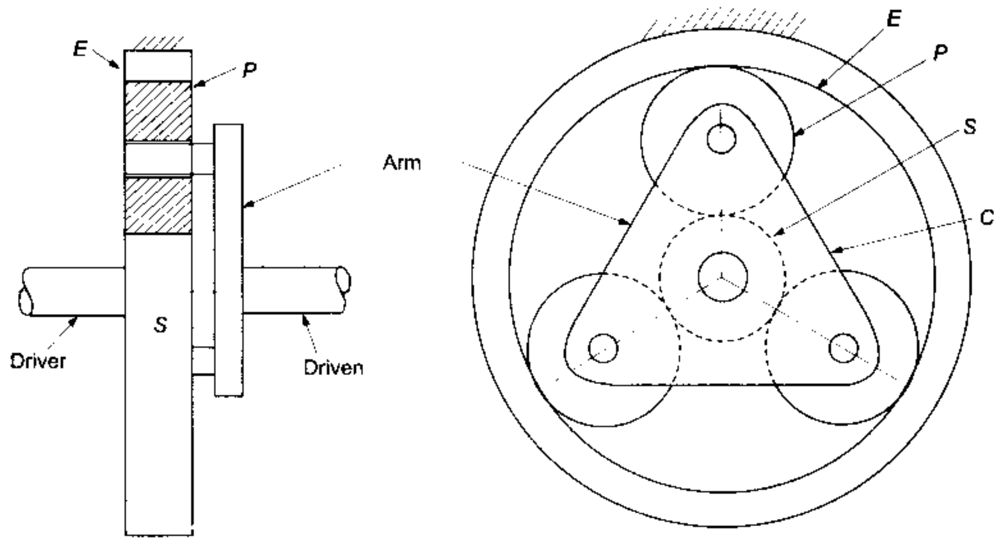


Fig.12.14 Epicyclic gear train

Now $$y = 1$$

$$x + y = 5$$

or $$x = 4$$

Gear E is stationary, therefore

$$-x\frac{z_s}{z_E} + y = 0$$

or $$-4\left(\frac{z_S}{z_E}\right) + 1 = 0$$

or $$\frac{z_s}{z_E} = \frac{1}{4}$$

Let $$z_S = 18$$

Then $$z_E = 72$$

Also $$d_S + 2d_P = d_E$$

Assuming same module for all gears. we have

$$z_S + 2z_P = z_E$$

or $$18 + 2z_P = 72$$

or $$z_P = 27$$

(b) $$T_S\omega_S = T_C\omega_C$$

$$120\omega_S = T_C w_C$$

or $$T_C = 120 \times 5 = 600 \text{ Nm}$$

Torque required to keep the annular gear stationary = 600 − 120 = 480 Nm

Example 12.11

In a reverted epicyclic gear train shown in Fig.12.15, the arm *F* carries two wheels *A* and *D* and a compound wheel *B*, *C*. The wheel *A* meshes with wheel *B* and the wheel *D* meshes with wheel *C*. $z_A = 80$, $z_D = 48$ and $z_C = 72$. Find the speed and direction of wheel *D* when wheel *A* is fixed and arm *F* makes 240 rpm clockwise.

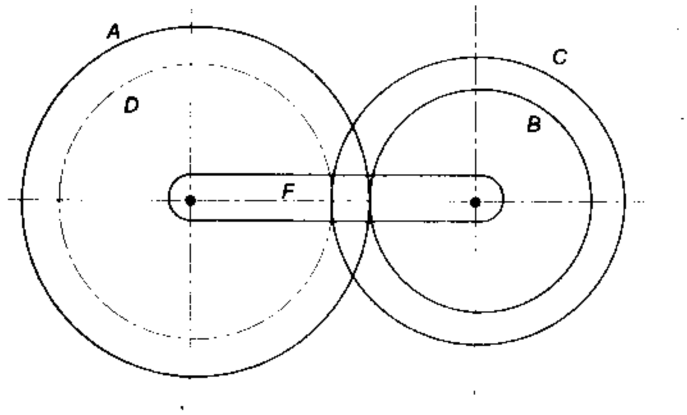


Fig.12.15 Reverted epicyclic gear train

Solution

$$z_A + z_B = z_C + z_D$$

$$z_B = 72 + 48 - 80 = 40$$

Table 12.12 can be used to find the speed of wheel *D*.

Table 12.12

Operation	Arm *F*	Wheel *A* 80	Compound wheel *B*, *C*, 40,72	Wheel *D* 48
1. Arm *F* fixed,	0	+1	$-\frac{z_A}{z_B} = \frac{-80}{40}$	$\frac{z_A}{z_B} \times \frac{z_C}{z_D}$
+1 revolution			$= -2$	$= \frac{80}{40} \times \frac{72}{48}$
to *A* ccw				$= 3$
2. Multiply by x	0	$+x$	$-2x$	$3x$
3. Add y	y	$y + x$	$y - 2x$	$y + 3x$

$$y = -240$$

$$x + y = 0$$

$$x = -y = 240 \text{ rpm}$$

$$N_D = y + 3x = -249 + 3 \times 240 = 480 \text{ rpm ccw}$$

Example 12.12

An epicyclic gear train shown in Fig.12.16 is composed of a fixed annular wheel *A* having 150 teeth. $z_B = 25$, $z_D = 40$ and *C* is an idle gear. Gear *D* is concentric with gear *A*. Wheels *B* and *C* are carried on an arm *E* which revolves clockwise at 120 rpm about the axis of *A*. Find the number of teeth of gear *C* and its speed and sense of rotation.

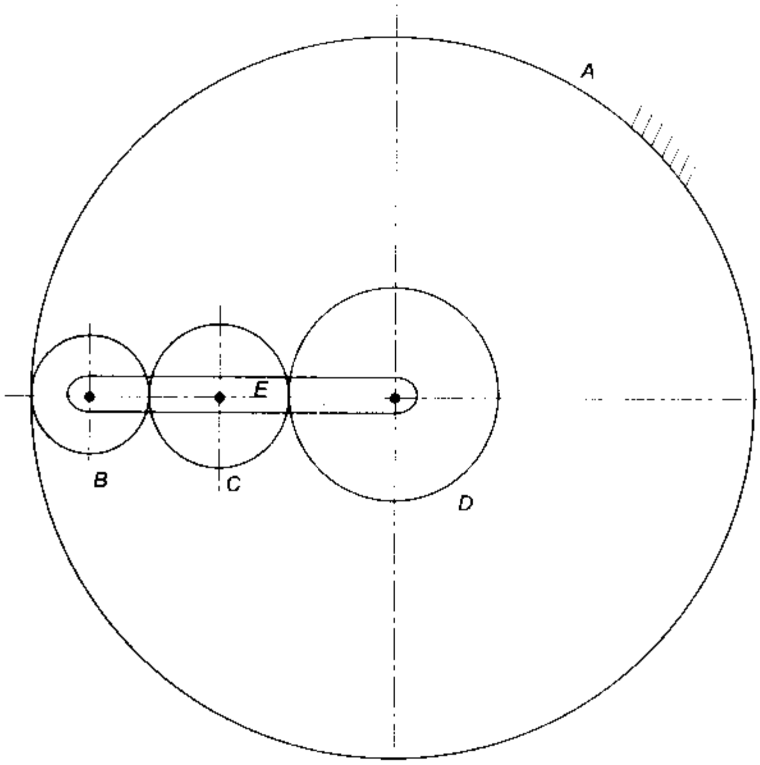


Fig.12.16 Epicyclic gear train

■ Solution

$$\frac{d_A}{2} = d_B + d_c + \frac{d_D}{2}$$

$$\frac{z_A}{2} = z_B + z_c + \frac{z_D}{2}$$

$$\frac{150}{2} = 25 + z_c + \frac{40}{2}$$

$$z_C = 75 - 25 - 20 = 30$$

Table 12.13 can be used to find the speed of gear *C*.

Table 12.13

Operation	Arm *E*	Gear *A*, 150	Gear *B*, 25	Gear *C*, 30	Gear *D*, 40
1. Arm *E* fixed, +1 revolution given to wheel *A* ccw.	0	+1	$+\frac{z_A}{z_B} = +\frac{150}{25} = +6$	$-\frac{z_A}{z_C} = -\frac{150}{30} = -5$	$+\frac{z_A}{z_D} = +\frac{150}{40} = +\frac{15}{4}$
2. Multiply by x	0	$+x$	$+6x$	$-5x$	$+15x/4$
3. Add y	y	$y+x$	$y+6x$	$y-5x$	$y+\frac{15x}{4}$

$$y = -120$$
$$x + y = 0; \quad x = 120$$
$$N_C = -5x + y = -5 \times 120 - 120 = -720 \text{ rpm i.e. } 720 \text{ rpm cw}$$

Example 12.13

For the compound epicyclic gear train shown in Fig.12.17, $z_A = 60$, $z_B = 40$, and $z_C = 25$. Find z_D and the speed of shaft connected to arm *E*, if the speed of shaft connected to sun gear is 120 rpm ccw and gear *D* is fixed.

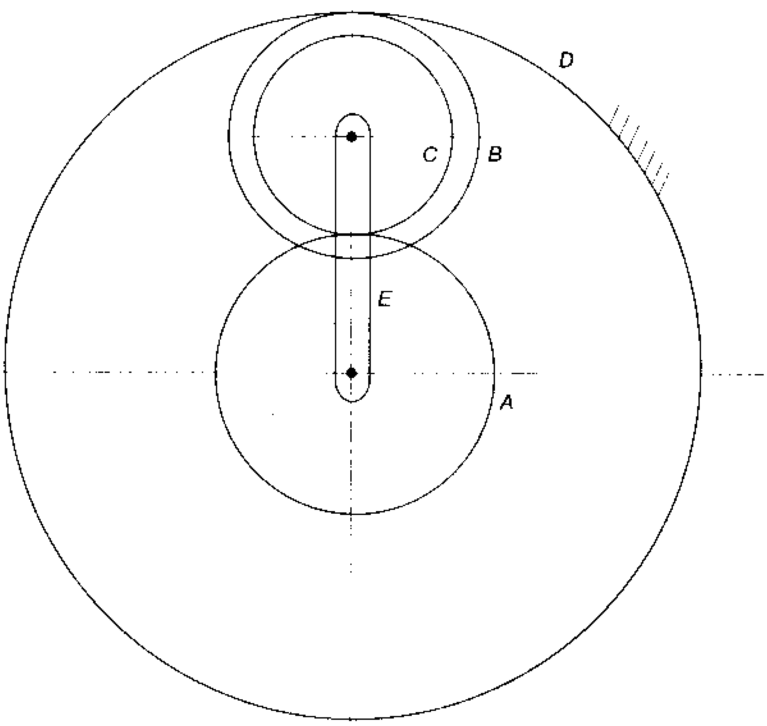


Fig.12.17 Compound epicyclic gear train

■ **Solution**

$$d_D = d_A + d_C + d_B$$
$$z_D = z_A + z_C + z_B$$
$$= 60 + 25 + 40 = 125$$

Table 12.14 can be used to find the speed of shaft connected to arm *E*.

Table 12.14

Operation	Arm *E*	Gear *A*, 60	Gears *B*, *C* 40, 25	Gear *D*, 40
1. Arm *E* fixed, +1 revolution given to wheel *A* ccw.	0	+1	$-\frac{z_A}{z_C}$ $= -\frac{60}{25}$ $= -2.4$	$-\frac{z_A}{z_C} \times \frac{z_B}{z_D}$ $= -\frac{60}{25} \times \frac{40}{125}$ $= -0.768$
2. Multiply by x	0	$+x$	$-2.4x$	$-0.768x$
3. Add y	y	$y + x$	$y - 2.4x$	$y - 0.768x$

$$y - 0.768x = 0$$
$$x + y = 0$$
$$x = 67.87 \text{ rpm}$$
$$y = 52.13 \text{ rpm ccw}$$

Example 12.14

Fig.12.18 shows an epicyclic speed reduction gear. The driving shaft is attached to arm *E*. The arm carries a pin on which the compound gear *B*, *C* is free to revolve. The gear *A* is keyed to the driven shaft and gear *D* is a fixed gear. $z_A = 24$, $z_B = 27$, $z_C = 30$ and $z_D = 21$. Determine (a) speed of driven shaft if driving shaft is rotating at 900 rpm counter-clockwise and (b) resisting torque on driven shaft and holding torque on gear *D*, if the input torque to driving shaft is 15 Nm.

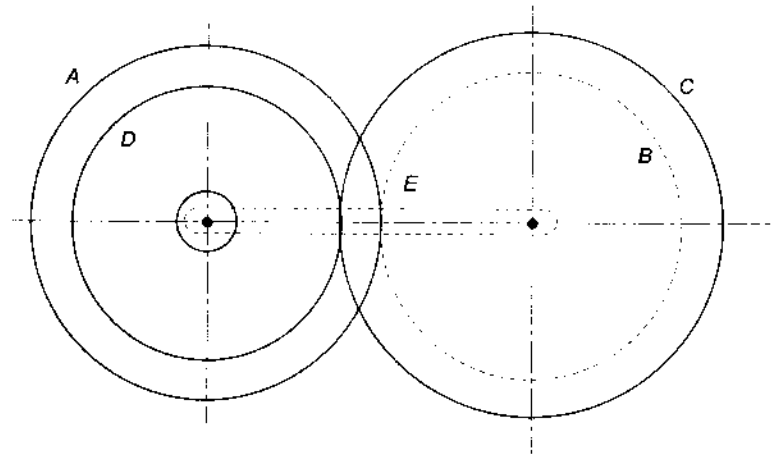


Fig.12.18 Epicyclic speed reduction gear

■ Solution

Table 12.15 can be used to find the speed of the gears.

Table 12.15

Operation	Arm E	Gear A, 60	Gears B, C 40, 25	Gear D, 40
1. Arm E fixed, +1 revolution given to wheel A ccw.	0	+1	$\frac{-z_A}{z_B} = -\frac{24}{27} = -\frac{8}{9}$	$\frac{-z_A}{z_B} \times \left(\frac{-z_C}{z_D}\right) = -\left(\frac{8}{9}\right) \times \left(-\frac{30}{21}\right) = \frac{80}{63}$
2. Multiply by x	0	$+x$	$-\frac{8x}{9}$	$\frac{80x}{63}$
3. Add y	y	$y + x$	$y - \frac{8x}{9}$	$y + \frac{80x}{63}$

(a)

$$N_D = 0,$$
$$y + \frac{80x}{63} = 0$$
$$N_E = y = 900$$
$$900 + \frac{89x}{63} = 0$$
$$x = -708.75$$
$$N_A = x = y = -708.75 + 900 = 191.25 \text{ rpm ccw}$$

(b)

$$M_1 + M_2 + M_3 = 0$$
$$M_1\omega_1 + M_2\omega_2 + M_3\omega_3 = 0$$

For the fixed member, $M_3 = 0$

$$M_1\omega_1 + M_2\omega_2 = 0$$

Resisting torque, $M_2 = \frac{-M_1\omega_1}{\omega_2} = -15 \times \frac{900}{191.25} = -70.59 \text{ Nm}$

Holding torque, $M_3 = -M_1 - M_2 = -15 - (-70.59) = 55.59 \text{ Nm}$

Example 12.15

In Fig.12.19, pinion A having 15 teeth is fixed to motor shaft. $z_B = 20$, $z_C = 15$, where B and C are a compound gear wheel. Wheel E is keyed to the machine shaft. Arm F rotates about the same shaft on which A is fixed and carries the compound wheel B, C. If the motor runs at 1200 rpm counter-clockwise, find (a) the speed of the machine shaft, (b) the torque exerted on the machine shaft if the motor develops a torque of 1200 Nm with an efficiency of 95%, and (c) ratio of the reduction gear.

■ Solution

$$d_D = 2d_B + d_A$$
$$z_D = 2z_B + z_A = 2 \times 20 + 15 = 55$$
$$d_E = d_A + d_B + d_C$$
$$z_E = z_A + z_B + z_C = 15 + 20 + 15 = 50$$

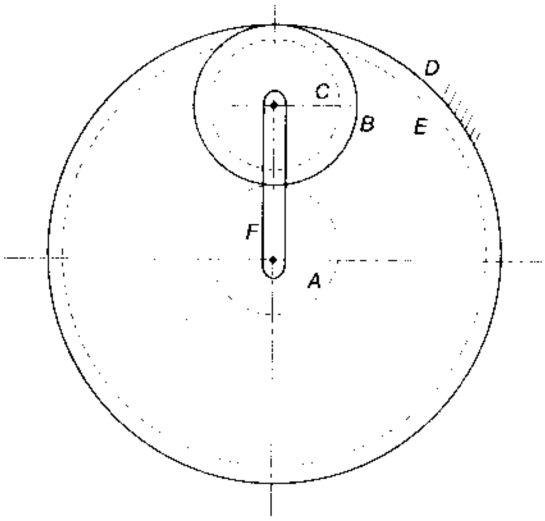


Fig.12.19 Mechanism with a compound gear wheel

Table 12.16 can be used to find the speed of the gears.

Table 12.16

Operation	Arm *F*	Gear *A*, 15	Gears *B*, *C* 20, 15	Gear *E*, 50	Gear *D*, 55
1. Arm *F* fixed, +1 revolution given to wheel *A* ccw.	0	+1	$\frac{-z_A}{z_B} = -\frac{15}{20} = -\frac{3}{4}$	$\frac{-z_A}{z_B} \times \left(\frac{z_C}{z_E}\right) = -\left(\frac{3}{4}\right) \times \left(\frac{15}{50}\right) = -\frac{9}{40}$	$\frac{-z_A}{z_B} \times \frac{z_B}{z_D} = \frac{-z_A}{z_D} = -\frac{15}{55} = -\frac{3}{11}$
2. Multiply by x	0	$+x$	$-\frac{3x}{4}$	$-\frac{9x}{40}$	$-\frac{3x}{11}$
3. Add y	y	$y + x$	$y - \frac{3x}{4}$	$y - \frac{9x}{40}$	$y - \frac{3x}{11}$

(a)

$$N_A = x + y = 1200$$

$$N_D = y - \frac{3x}{11}$$

$$x = 942.86 \text{ rpm}$$

$$y = 257.14 \text{ rpm}$$

$$N_E = y - \frac{9x}{40} = 257.14 - 9 \times \frac{942.86}{40} = 45 \text{ rpm}$$

(b)

$$M_2 = -M_1\omega_1 \frac{\eta}{\omega_2} = -120 \times 1200 \times \frac{0.95}{45} = -3040 \text{ Nm}$$

(c)

$$\text{Ratio of reduction gear} = \frac{N_A}{N_E} = \frac{1200}{45} = 26.67$$

Example 12.16

An epicyclic gear train consists of sun wheel S, a fixed internal gear E and three identical planet wheels P carried on a star-shaped planet carrier C, as shown in Fig.12.20. The planets rotate at 1/5th of the speed of sun wheel. The minimum number of teeth on any wheel is 18. The driving torque on the sun wheel is 120 N m. Find (a) number of teeth on different wheels of the train and (b) torque necessary to keep the internal gear stationary.

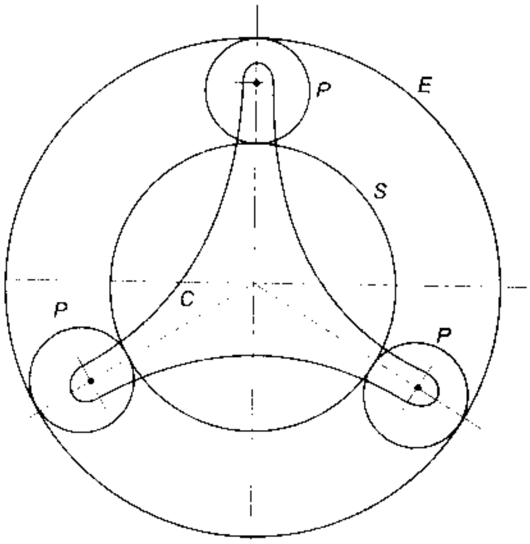


Fig.12.20 Epicyclic gear train

■ Solution

Table 12.17 can be used to find the speed of the gears.

Table 12.17

Operation	Planet carrier C	Sun gear S	Planet gear P	Annular gear E
1. Planet carrier fixed, +1 revolutions to S, ccw	0	+1	$\frac{-z_S}{z_P}$	$\frac{-z_S}{z_E}$
2. Multiply by x	0	$+x$	$\frac{-z_S}{z_P} \cdot x$	$\frac{-z_S}{z_E} \cdot x$
3. Add y	y	$x + y$	$\frac{-z_S}{z_P} \cdot x + y$	$\frac{-z_S}{z_E} x + y$

$$N_C = \frac{N_S}{5}$$

$$y = \frac{(x + y)}{5}$$

or

$$x = 4y$$

$$N_E = 0$$

$$\frac{-z_s}{z_E} \cdot x + y = 0$$

$$y - 4y\left(\frac{z_S}{z_E}\right) = 0$$

$$\frac{z_S}{z_E} = \frac{1}{4}$$

Let $z_S = 18$, then $z_E = 72$

$$d_E = d_S + 2d_P$$

$$z_E = z_S + 2z_P$$

$$72 = 18 + 2z_P$$

$$z_P = 27$$

(b)

$$M_2 = -M_1\frac{\omega_1}{\omega_2} = -5M_S = -5 \times 120 = -600 \text{ Nm}$$

$$M_3 = -M_1 - M_2 = -120 + 600 = 480 \text{ Nm}$$

Example 12.17

Fig.12.21 shows the arrangement of wheels in a compound epicyclic gear train. The sun wheel S_2 is integral with the annular wheel A_1. The two arms are also integral with each other. $z_{S1} = z_{S2} = 24$, $z_{A1} = z_{A2} = 96$.

(a) If the shaft X rotates at 2000 rpm, find the speed of shaft Y, when A_2 is fixed.

(b) At what speed does Y rotate when A_2 rotates at 200 rpm, in the same direction as S_1, which is rotating at 2000 rpm.

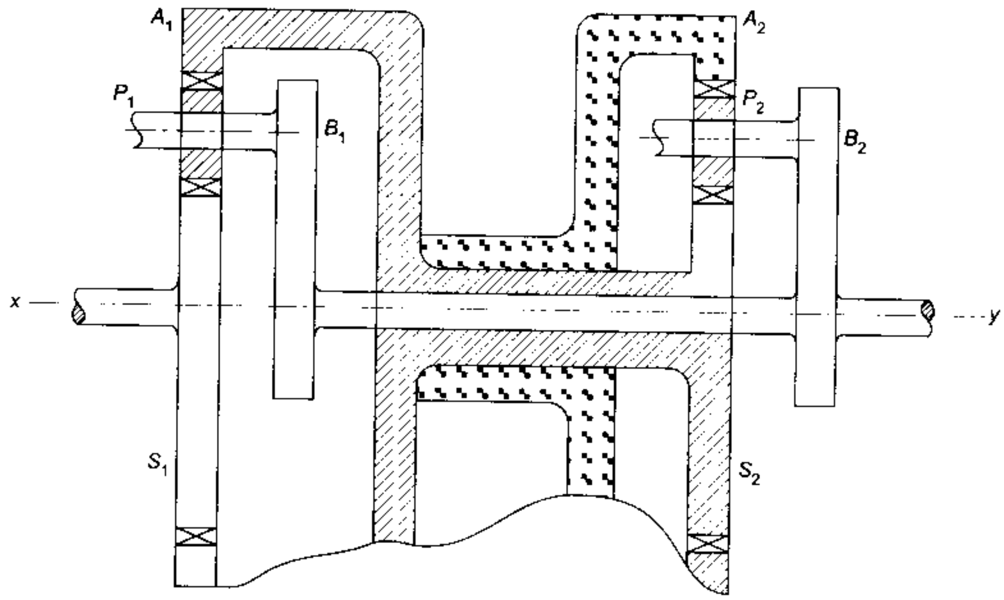


Fig.12.21 Compound epicyclic gear train

■ Solution

Table 12.18 can be used to find the speed of the gears.

Table 12.18

Operation	Arm	S_1	P_1	A_1	S_2	P_2	A_2
1. Fix arm B_1, +1 revolution to S_1 ccw	0	+1	$\frac{-24}{z_{P1}}$	$\frac{-24}{96} = -1/4$	$\frac{-24}{96} = -1/4$	$\frac{+24}{96} \times \frac{24}{z_{P2}} = 6/z_{P2}$	$\frac{+24}{96} \times \frac{24}{96} = \frac{+1}{16}$
2. Multiply by x	0	$+x$	$\frac{-24x}{z_{P1}}$	$\frac{-x}{4}$	$\frac{-x}{4}$	$\frac{6x}{z_{P2}}$	$\frac{+x}{16}$
3. Add y	y	$y+x$	$\frac{-24x}{z_{P1}} + y$	$\frac{-x}{4} + y$	$\frac{-x}{4} + y$	$\frac{6x}{z_{p2}} + y$	$\frac{+x}{16} + y$

(a) A_2 fixed,

$$\frac{x}{16} + y = 0$$
$$x = -16y$$
$$x + y = 2000$$
$$y = -\frac{400}{3} \text{ rpm}$$

Speed of shaft
$$Y = y = \frac{400}{3} \text{ rpm cw}$$

(b)
$$A_2 = +200 \text{ rpm}$$
$$\frac{x}{16} + y = 200$$
$$x + y = 2000$$
$$x = 1920; \quad y = 80 \text{ rpm ccw}$$

Exercises

1 In a reverted gear train, as shown in Fig.12.22, two shafts A and B are in the same straight line and are geared together through an intermediate parallel shaft C. The gears connecting the shafts A and C have a module of 3 mm and those connecting the shafts C and B have a module of 4.5 mm. The speed of shaft A is to be greater than 12 times the speed of shaft B. The ratio of each reduction is the same. Find suitable number of teeth on all gears. The minimum number of teeth is 18. Also find the exact velocity ratio and the distance of shaft C from A and B.

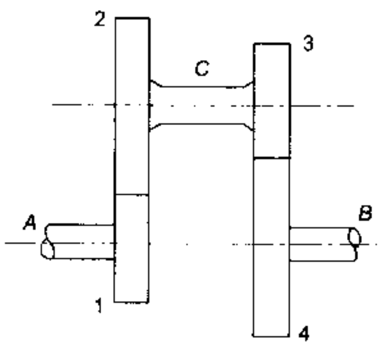


Fig.12.22 Reverted gear train

2 In an epicyclic gear train, as shown in Fig.12.23, the number of teeth on wheels *A*, *B* and *C* are 50, 25 and 52 respectively. If the arm rotates at 420 rpm clockwise, find (a) speed of wheel *C* when *A* is fixed, and (b) speed of wheel *A* when *C* is fixed.

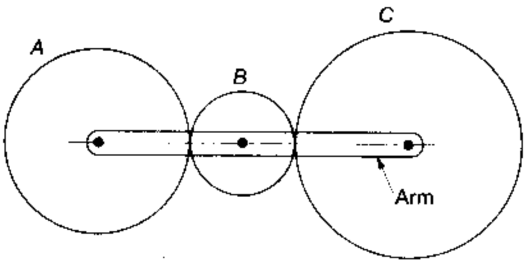


Fig.12.23 Epicyclic gear train

3 An epicyclic gear train, as shown in Fig.12.24, is composed of a fixed annular wheel *A* having 150 teeth. The wheel *A* is meshing with wheel *B* which drives wheel *D* through an idle wheel *C*, *D* being concentric with *A*. The wheels *B* and *C* are carried on an arm which revolves clockwise at 100 rpm about the axis of *A* and *D*. If the wheels *B* and *D* have 25 teeth and 40 teeth respectively, find the number of teeth on *C* and the speed and sense of rotation of *C*.

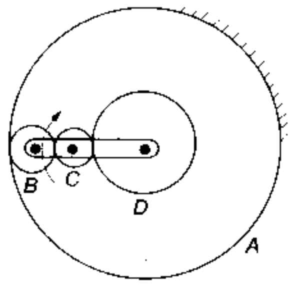


Fig.12.24 Epicyclic gear train

4 An epicyclic gear train, as shown in Fig.12.25, has a sun wheel *S* of 30 teeth and two planet wheels *P* of teeth 50 each. The planet wheels mesh with the teeth of internal gear *a*. The driving shaft carrying the sun wheel transmits 6 kW at 300 rpm. The driven shaft is connected to an arm which carries the planet wheels. Determine the speed of the driven shaft and the torque transmitted, if the overall efficiency is 95%.

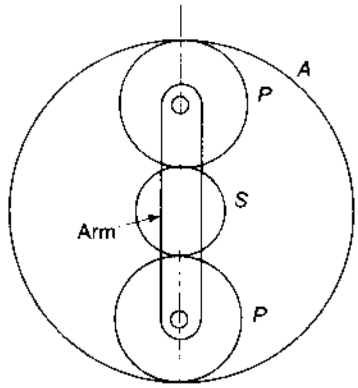


Fig.12.25 Epicyclic gear train

5 A compound epicyclic gear train is shown in Fig.12.26. The gears *A*, *D* and *E* are free to rotate on the axis *P*. The compound gears *B* and *C* rotate together on the axis *Q* at the end of arm *F*. All the gears have equal module. The number of teeth on the gears *A*, *B* and *C* are 18, 45 and 21 respectively. The gears *D* and *E* are annular gears. The gear *A* rotates at 120 rpm counter-clockwise and the gear *D* rotates at 450 rpm clockwise. Find the speed and direction of the arm and the gear *E*.

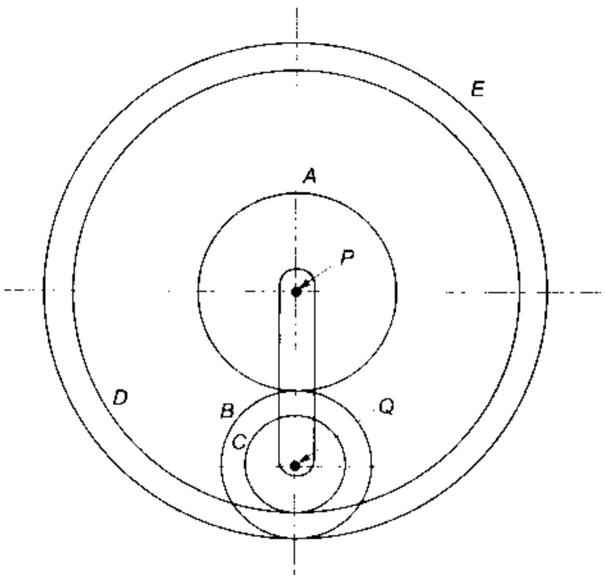


Fig.12.26 Compound epicyclic gear train

6 An epicyclic gear train shown in Fig.12.27, consists of two sun wheels A and *D* with 28 and 24 teeth respectively, engaged with a compound planet wheels *B* and *C* with 22 and 26 teeth. The sun wheel *D* is keyed to the driven shaft and the sun wheel *A* is a fixed wheel coaxial with the driven shaft. The planet wheels are carried on an arm *E* from the driving shaft which is coaxial with the driven shaft. Find the velocity ratio of gear train. If 1.2 kW is transmitted and input speed is 120 rpm, determine the torque required to hold the sun wheel *A*.

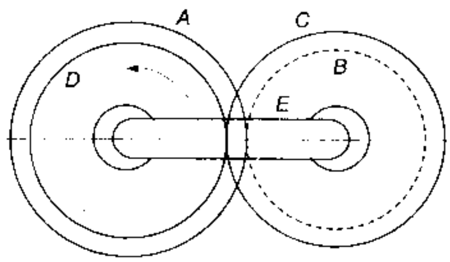


Fig.12.27 Epicyclic gear train

7 (a) Give a list of the common applications of planetary gear trains. Describe the working of the differential mechanism of a motor car.

(b) In the planetary gear train shown in Fig.12.28, gear 1 has 50 teeth and gear 3 has 90 teeth. Determine the number of equally spaced planets that can be used without overlapping. The gears are standard. The formula used is to be derived, stating the assumptions made.

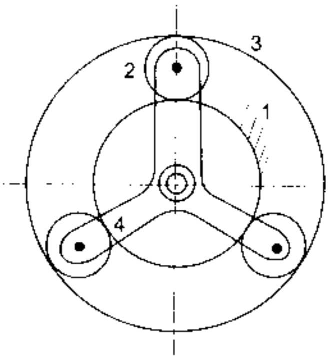


Fig.12.28 Planetary gear train

8 An epicyclic gear train is shown in Fig.12.29. The main driving shaft *G* has a gear S_1 integrally mounted and driving the internal gear A_1 on the casing through two intermediate gears P_1 mounted on either side. The gears P_1 are free to revolve on arms R_1 which are integral with gear S_2 which in turn drives the internal gear A_2 on another casing through two gears P_2. The driven shaft *H* is integral with the casing carrying the internal gear A_1 and arms R_2 on which the gears P_2 are free to rotate. The casing A_2 and gear S_2 are free to rotate on shaft *G*.

Calculate the speed of shaft *H* when *G* rotates at 1000 rpm counter-clockwise when (a) A_2 is stationary; (b) A_2 rotates at 500 rpm clockwise.

The number of teeth on gears are: $S_1 = S_2 = 30$, $A_1 = A_2 = 90$.

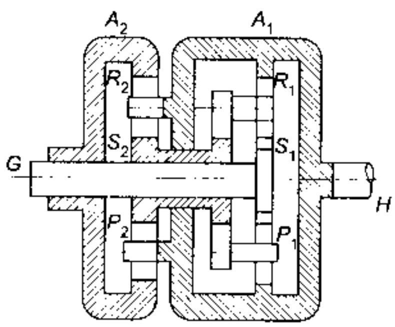


Fig.12.29 Epicyclic gear train

9 Figure 12.30 shows diagramatically a compound epicyclic gear train. Wheels *A*, *D* and *E* are free to rotate independently on spindle *C*, while *B* and *C* are compound and rotate together on spindle *P*, on the end of arm OP. All the teeth on different wheels have the same module. A has 12 teeth, *B* has 30 teeth and *C* has 14 teeth cut externally. Find the number of teeth on wheels *D* and *E* which are cut internally.

If the wheel a is driven clockwise at 1 rps while *D* is driven clockwise at 5 rps, determine the magnitude and direction of the angular velocities of arm *OP* and wheel *E*.

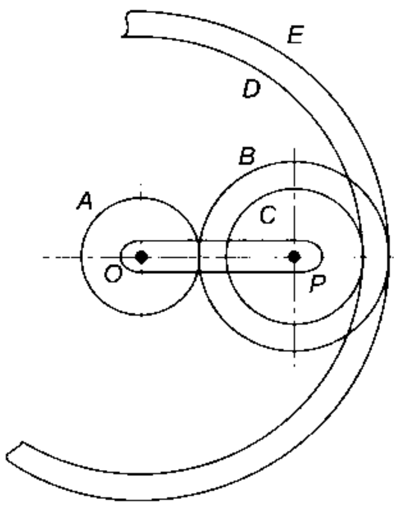


Fig.12.30 Compound epicyclic gear train

10 A mechanism for recording the distance covered by the bicycle, as shown in Fig.12.31, is as follows:

There is a fixed annular wheel A of 22 teeth and another annular wheel B of 23 teeth, which rotates loosely on the axis of A. An arm driven by the bicycle wheel through gearing not described, also revolves freely on the axis of A and carries on a pin at its extremity two wheels C and D, which are integral with one another. The wheel C has 19 teeth and meshes with A and the wheel D with 20 teeth meshes with B. The diameter of the bicycle wheel is 0.7 m. What must be the velocity ratio between the bicycle wheel and the arm, if B makes one revolution per 1.5 km covered?

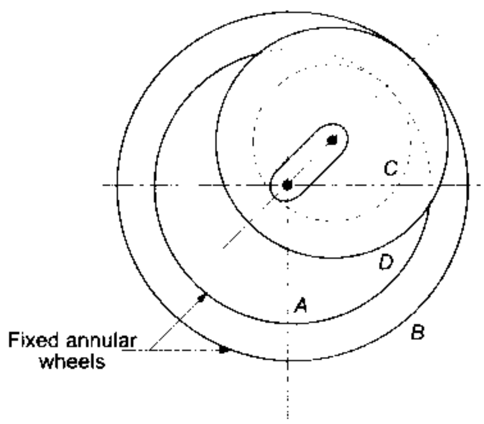


Fig.12.31 Mechanism for recording the distance covered by a bicycle

11 In the epicyclic gear train shown in Fig.12.32, the arm A, carrying the compound wheels D and E, turns freely on the output shaft. The input speed is 1000 rpm in counter-clockwise direction when seen from the right. Input power is 7.5 kW. Calculate the holding torque to keep the wheel C fixed. The number of teeth for different gears are as shown in the figure.

12 In the epicyclic gear train shown in Fig.12.33, the compound wheels *E* and *F* rotate freely on shaft *A* which carries the planet carrier *G*. The planets *B* and *C* are compounded gears. The number of teeth on each gear are: $z_E = 30$, $z_H = 15$, $z_B = 20$, $z_C = 18$, $z_D = 68$.

The shafts *A* and *K* rotate in the same direction at 250 rpm and 100 rpm respectively. Determine the speed of shaft J.

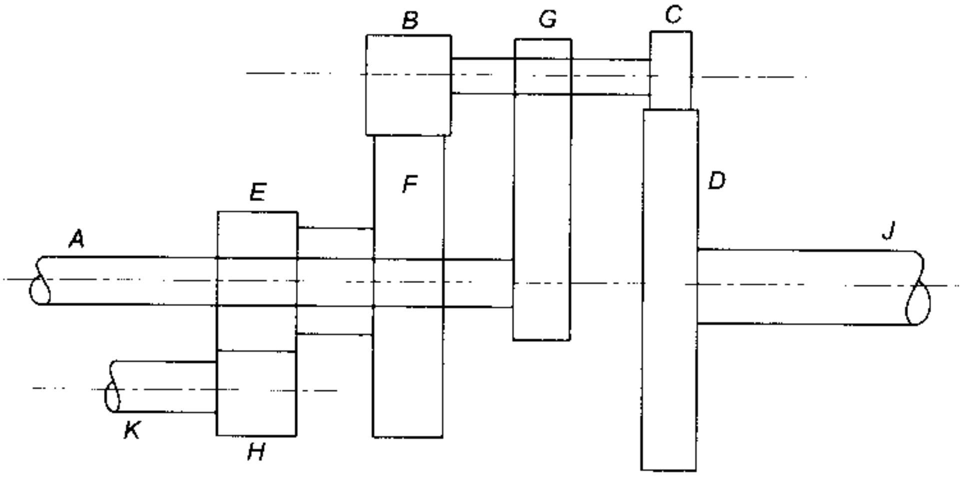


Fig.12.32 Epicyclic gear train

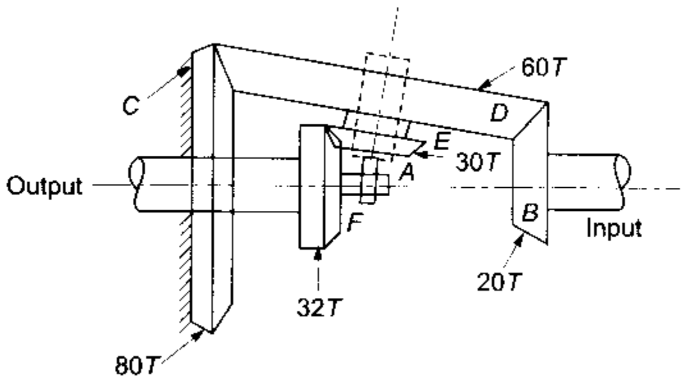


Fig.12.33 Epicyclic gear train

13 The speed ratio of the reverted gear train shown in Fig.12.34 is 12. The module of gears *A* and *B* is 3.125 mm and of gears *C* and *D* is 2.5 mm. Calculate the suitable number of teeth for the gears. No gear is to have less than 24 teeth.

14 In an epicyclic gear train of the sun and planet type shown in Fig.12.35, the pitch circle diameter of the annular wheel *A* is to be nearly equal to 216 mm, and the module is 4 mm. When the annular wheel is stationary, the spider which carries three planet gears *P* of equal size, has to make one revolution for every five revolutions of the driving spindle carrying *S* gear. Determine the number of teeth on all the wheels and also the exact pitch circle diameter of *A*.

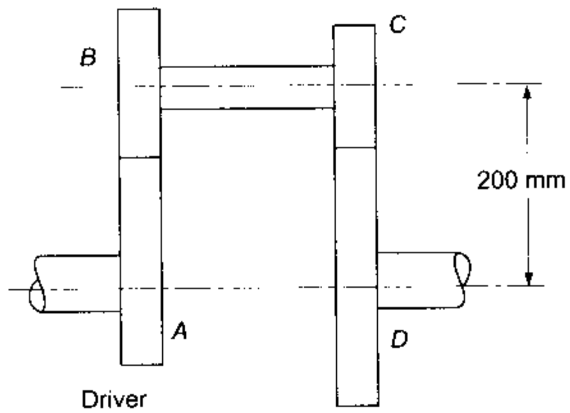


Fig.12.34 Reverted gear train

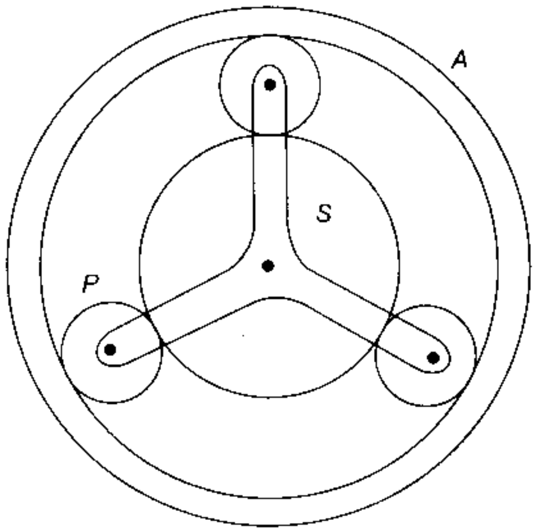


Fig.12.35 Epicyclic gear train of the sun and planet type

15 An epicyclic gear train as shown in Fig.12.36, has a sun wheel *S* of 30 teeth and two planet wheels *P–P* of 45 teeth. The planet wheels mesh with the internal teeth of a fixed annulus *A*. The driving shaft carrying the sun wheel transmits 4 kW at 360 rpm. The driven shaft is connected to an arm, which carries the planet wheels. Determine the speed of the driven shaft and the torque transmitted, if the overall efficiency is 95%.

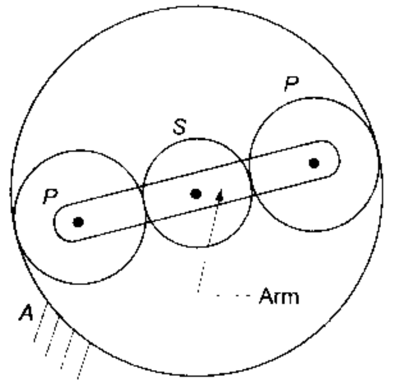


Fig.12.36 Epicyclic gear train

16 Two shafts X and Y are in the same line (axes in one line). They are geared together through an intermediate parallel shaft carrying wheels B and C which mesh with the wheels A and D respectively, as shown in Fig.12.37. Wheels A and B have a module of 4 mm and the wheels C and D have a module of 9 mm. The number of teeth on any wheel is not to be less than 15 and the speed of D is to be about, but not greater than 1/12th the speed of A and the ratio of each reduction is the same. Find (a) suitable number of teeth for the wheels, (b) the actual reduction and (c) the distance of the intermediate shaft from the axes of the shafts X and Y (centre distance).

How is addendum modification related to correction of gears and when are they used in practice? (Gear A is on shaft X and gear D on shaft Y)

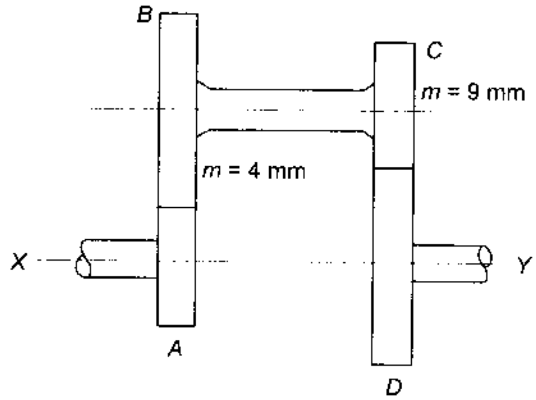


Fig.12.37

17 Fig.12.38 shows a compound epicyclic gear train in which two sun gears S_1 and S_2 are integral with the input shaft X. The arm B_2 is integral with the output shaft Y. The planet gear P_2 revolves on a pin attached to arm B_2 and meshes with gear S_2 and annular gear A_2. The annular gear A_2 is coaxial with the input shaft. The planet gear P_1 meshes with the fixed annular gear A_1 and sun gear S_1. The gear P_1 revolves on a pin fixed to gear A_2. The number of teeth on gears are: $S_1 = 26$, $S_2 = 31$, $A_1 = 88$, $A_2 = 83$. The input power on shaft X is 10 kW at 1000 rpm cccw. Find (a) the speed and torque at shaft Y, assuming efficiency of 96% and (b) the torque required to hold the gear A_1 stationary.

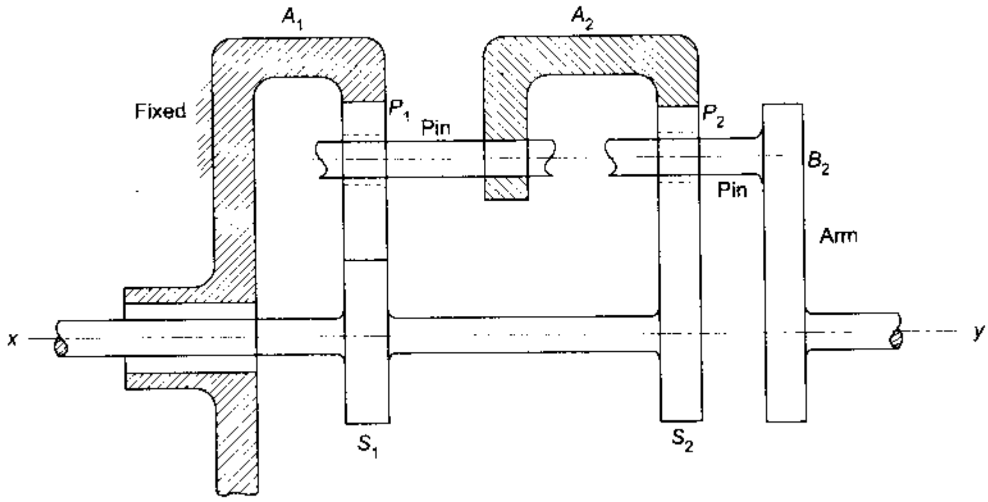


Fig.12.38